

# Supersymmetric Microscopic Theory of the Standard Model

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## Abstract

We promote the microscopic theory of standard model (MSM) [1,2] into supersymmetric framework in order to solve its technical aspects of vacuum zero point energy and hierarchy problems, and attempt, further, to develop its realistic viable minimal SUSY extension. Among other things that - the MSM provides a natural unification of geometry and the field theory, has clarified the physical conditions in which the geometry and particles come into being, in microscopic sense enables an insight to key problems of particle phenomenology and answers to some of its nagging questions - a present approach also leads to quite a new realization of the SUSY yielding a physically realistic particle spectrum. It stems from the special subquark algebra, from which the nilpotent supercharge operators are derived. The resulting theory makes plausible following testable implications for the current experiments at LEP2, at the Tevatron and at LHC drastically different from those of the conventional MSSM models:

- *All the particles and the Higgs bosons never could emerge in spacetime continuum, thus, they cannot be discovered in any experiment nor at any energy range.*
- *For each of the three SM families of quarks and leptons there are corresponding heavy family partners with the same quantum numbers <sup>1</sup> and common mass-shift coefficients  $(1+k)$  given for the low-energy poles  $k_1 > \sqrt{2}$ ,  $k_2 = \sqrt{8/3}$  and  $k_3 = 2$ , lying far above the electroweak scale, respectively, at the energy threshold values:  $E_1 > (419.6 \pm 12.0)\text{GeV}$ ,  $E_2 = (457.6 \pm 13.2)\text{GeV}$  and  $E_3 = (521.4 \pm 15.0)\text{GeV}$ .*

## 1 Introduction

A phenomenological standard model (SM) of high energy physics [4-22] with enormous success settles order in entangled experimental data. Although it has proven to be in spectacular agreement with experimental measurements and highly successful in a description and predicting a wide range of phenomena, however, it suffers from some vexing problems and many key questions of both the phenomenological and SUSY aspects have yet to be answered.

- In phenomenological aspect the mechanism of the electroweak symmetry breaking is a complete mystery. The most problematic ingredient of such a breaking is the Higgs boson (in simplest version), which has not yet been discovered experimentally. If a weakly interacting Higgs boson exists, it will then appear below the TeV scale. Many possible

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<sup>1</sup>This prediction was directly ensued from the MSM [1,2], as well the similar one was made in phenomenological consideration by S.L.Adler [3]).

extension of the Higgs sector have been considered (see e.g. [20,21,23,24] and references therein). Along with the verifying experimentally of the Higgs sector, the most important open questions of the SM are as follows: We have no understanding why the SM is as it is? Why is the gauge symmetry? Why is this the particle spectrum? Why the electroweak symmetry breaking sector consists of just one  $SU(2)_L$  doublet of Higgs bosons as it is in SM? The untested aspects of SM are the mass spectrum of the particles, the mixing patterns and the CP violation. The latter is introduced through complex Yukawa couplings of fermions to Higgs bosons, resulting in complex parameters in the CKM matrix. The SM contains a large number of arbitrary parameters, while a consistent complete theory would not have so many free parameters.

- There is another line of reasoning which supports the side of supersymmetrization of the SM, i.e., there are two well known principal issues which remain open in the SM. The first is the vacuum zero point problem standing before any quantum field theory. Second is often referred to as the problem of quadratic divergences or the hierarchy and naturalness problem (the dimensional analysis problem) arisen as the quadratic growth of the Higgs boson mass beyond tree level in perturbation theory, namely, the extreme difference in energy scales in the theory is inconsistent in the fundamental scalar sector. This is strong indication for the physics beyond SM. These last two problems can be solved by extending the symmetry of the theory to supersymmetry [25-49], which is believed in conventional physics to be manifest at energies in the TeV range. Given the SUSY requiring doubling the number of all the particles by their SUSY partners (sparticles), the quantum radiative corrections may cancel because some loop diagrams vanish due to cancellation between bosons and fermions since they have opposite signs. Then, if the SUSY is present in the TeV range, the masses of the Higgs bosons are no more unstable than fermion masses, whose smallness is natural and hold due to the approximate chiral symmetries. In this manner, its simplest form, SUSY solves the technical aspects of the hierarchy problem as well as the zero point energy problem, when due to power of the boson-fermion cancellation the zero point energy of the fermions exactly cancels that of the bosons and the degeneracy is not arisen. Therefore, in usual, the SM should be regarded as an effective low energy field theory valid up to the energy range smaller than a few hundred GeVs.

- However, the SUSY in turn introduces its own set of difficulties. Despite the beautiful mathematical features of SUSY theories and that the SUSY has been theoretically invented almost three decades ago, but a physically realistic realization of SUSY had not been achieved yet and this principal problem was ever since much the same as now. In all suggested SUSY theories the supercharges have been inserted in *ad hoc* manner directly into the four-dimensional spacetime continuum adding a new structure, i.e., a new four odd fermionic dimensions. In fact, a physical essence of the basic concept of supercharge remains unknown and, therefore, the physical theory is beset by various difficulties. Perhaps the most discouraging and disturbing feature of the general class of proposed SUSY theories is the absence at the moment of a solid experimental motivation of supersymmetry, i.e., there is not a direct experimental evidence for the existence of any of the numerous new sparticles predicted by such theories. It is clear, then, that SUSY cannot be an exact symmetry in nature but has to be realized at least in broken phase. The last one is the least understood aspect of such theories. The spontaneously broken SUSY should be ruled out at once since it runs into phenomenological difficulties [31-49]. One of the viable way out from this situation is an explicit breaking of the global SUSY. A generic parametrization of this phenomenon introduces the much larger free parameter

space ( $\simeq 124$ ) in the models of minimal supersymmetric extension of the SM (MSSM-124 [32]). Thus, it is important to develop the other schemes that attempt to reduce the number of free parameters. The conventional SUSY theories predict that the sparticles must reside in the TeV range. All such arguments that nature is supersymmetric, and that SUSY is broken at scales not too different than the weak scale, are theoretical. The next generation of experiments at Fermilab and CERN [35,39,45,49-60] will explore this energy range, where at least some of sparticles are expected to be found.

- All this variety prompts us, further, to adopt the idea that perhaps a more deeper level of organization of physical world may be existed. In the light of current status of particle physics, any new more elaborated outlook seems worthy of investigation. To solve in microscopic sense some of the above mentioned problems of phenomenological aspect the MSM is built up in [1,2]. The operator manifold (OM) formalism (part I) [1] enables to develop an approach to the unification of the geometry and the field theory, as well the quantization of geometry different from all existing schemes. Here we explore the query how did the geometry and fields, as they are, come into being? In the first a major purpose is to prove our physical outlook embodied in the idea that the geometry and fields, with the internal symmetries and all interactions, as well the four major principles of relativity (special and general), quantum, gauge and colour confinement are derivative, and they come into being simultaneously (sec.2). The substance out of which the geometry and fields are made is the “primordial structures” involved into reciprocal “linkage” establishing processes (subsec.2.3, 2.4) We generalize this formalism via the concept of operator multimanifold (OMM) (sec.3), which yields the MW geometry (subsec.3.2) involving the spacetime continuum and internal worlds of given number. All this is not merely an exercise in abstract reasoning but presumably bears directly on the geometry of the universe in which we live. In an enlarged framework of the OMM we define and clarify the conceptual basis of subquarks and their characteristics stemming from the various symmetries of the internal worlds (subsec.3.2). The OMM formalism has the following features:

- *It provides a natural unification of the geometry- yielding the special and general relativity principles (subsec.2.2), and the fermion fields serving as the basis for the constituent subquarks (subsec.3.2).*

- *It has cleared up the physical conditions in which the geometry and particles come into being (subsec.2.2, 3.1).*

- *The subquarks emerge in the geometry only in certain permissible combinations utilizing the idea of the subcolour (subquark) confinement principle (subsec.3.2), and have undergone the transformations yielding the internal symmetries and gauge principle (subsec.2.6).*

We developed the MSM (part II) [2] based on the OMM formalism, which attempts to answer to some of the above mentioned questions of particle phenomenology. All the fields along with the spacetime component have nontrivial composite internal MW structure (subsec.12.1) such that the possible elementary particles are thought to be composite dynamical systems in analogy to quantum mechanical stationary states of compound atom, but, now a dynamical treatment built up on the MW geometry is quite different and more amenable to qualitative understanding. The microscopic structure of leptons, quarks and other particles will be governed by the only possible conjunctions of constituent subquarks implying concrete symmetries. Although within considered schemes the subquarks are defined on the internal worlds, however the resulting spacetime components of particles, which we are going to deal with to describe the leptons and quarks defined on the space-

time continuum, are affected by them (sec.12) in such a way that they carry exactly all the quantum numbers of the various constituent subquarks of the given composition. The hypothesis of existence of the MW structures manifests its virtue by solving some key problems of particle phenomenology, when we attempt to suggest a microscopic approach to the properties of particles and interactions. First of all the theoretical significance of the MSM resides in the microscopic interpretation of all physical parameters.

Continuing this program towards the supersymmetrization, in this article we shall attempt, further, to promote the MSM into the SUSY framework by elaborating its realistic manifestly minimal SUSY extension (M\$MSM) (sec.15). The major difference of outlined here supersymmetric approach (MW-SUSY) from those of conventional SUSY theories is as follows:

- *The MW-SUSY has an algebraic origin in the sense that it stems from a special subquark algebra defined on the internal worlds, while the nilpotent supercharge operators are derived (sec.4). Therefore, the MW-SUSY has realized only on the internal worlds, but not on the spacetime continuum, which are all the ingredients of the broken super-multimanifold (\$MM).*

- *Defined on the \$MM (sec.8) it implies the super-algebra different from the conventional SUSY algebra.*

- *Here we are led to the principal point of drastic change of the standard SUSY scheme to specialize the superpotential to be in such a form of eq.(11.28)-eq.(11.34), which allows within this framework, further, to build up the MSM (sec.12).*

We develop the microscopic approach to the isospinor Higgs boson with self-interaction and Yukawa couplings (subsec.12.9-sec.13), wherein the two complex self-interacting isospinor-scalar Higgs doublets ( $H_u, H_d$ ) as well as their spin- $\frac{1}{2}$  SUSY partners ( $\widetilde{H}_u, \widetilde{H}_d$ ) Higgsinos have arisen on the  $W$ -world as the Bose-condensate. In contrast to SM, the MSM predicts the electroweak symmetry breaking in the  $W$ -world by the vacuum expectation value (VEV) of spin zero Higgs bosons and its transmission from the  $W$ -world to the spacetime continuum (subsec.12.14). This is the most remarkable feature of suggested approach, especially, in the view of existing great belief of the conventional theories for a discovery of the Higgs boson with other new particles at next round of experiments at LEP2, at the Tevatron, at LHC and other colliders, which will explore the TeV energy range (e.g. see [50]). The LEP2 data (is currently running at 189 GeV) provide a lower limit  $m_H > 89.3\text{GeV}$  on its mass in simplest version. Furthermore, there is a tight upper limit ( $m_{h^0} < 150\text{GeV}$ ) on the mass of the lightest Higgs boson  $h^0$  among the 5 physical Higgs bosons predicted by the MSSM models. The current direct search limits from LEP2 give  $m_{h^0} > 75\text{GeV}$ . Therefore, the future searches for this boson (if the mass is below 150 GeV or so) would be a crucial point in testing the efforts made in the conventional models building as well in the present MSM based on a quite different approaches. Actually, reflecting upon the results far obtained in the sec.12, in strong contrast to conventional theories, the MSM rejects drastically any expectation of discovery of any Higgs boson, but in the same time it expects to include a rich spectrum of new particles at higher energies. Namely, if the MSM proves viable it becomes an crucial issue to hold in experiments the following two solid tests:

- *The Higgs bosons never could emerge in spacetime continuum since they have arisen only on the internal  $W$ -world, i.e., thus, the unobserved effects produced by such bosons cannot be discovered in experiments nor at any energy range.*

- *For each of the three SM families of quarks and leptons, there are corresponding heavy family partners with the same quantum numbers lying far above the electroweak scale.*

Regarding to the last phenomenological implication of the MSM, it is remarkable that the similar in many respects prediction is made in somewhat different context by S.L.Adler [3] within a phenomenological scheme of a compositeness of the quarks and leptons. It based on the generic group theoretical framework of rishon type models exploring the preon constituents. But, therein a present, a bit premature, state of the theory does not allow the exact estimate of this scale. Although one admits that such a scale could be much higher than electroweak scale, however, it is necessary special argumentation in support of validity of this prediction in a case if this scale has turned out to be low enough, namely, if these heavy partners lie not too far above the electroweak scale. Even thus, as it is notified in [3], one must not worry for the existence of 6 heavy flavors, which is then marginally compatible with the current LEP data [18]. A complete analysis of this question, naturally, is now possible in suggested microscopic approach. The MSM enables oneself to study in detail the phenomenology associated with such extra heavy families and to estimate the value of energy threshold of their creation (sec.12.14). While, the low energy scale could not be realized since it lies far below the energy threshold of the next pole for appearing of the heavy partners. The estimate gives the common mass-shift coefficients  $(1 + k)$ , where  $k$  reads for the next few low energy poles with respect to the lowest one:  $k_0 = 0$ ,  $k_1 > \sqrt{2}$ ,  $k_2 = \sqrt{8/3}$  and  $k_3 = 2$ . The first one obviously does not produce the extra families, but the energy thresholds corresponded to the next non-trivial poles can be respectively written:  $E_1 > (419.6 \pm 12.0)GeV$ ,  $E_2 = (457.6 \pm 13.2)GeV$  and  $E_3 = (521.4 \pm 15.0)GeV$ .

These predictions above together with a new one given in the sec.17 that

- *the sparticles could never emerge in spacetime continuum since they have arisen only on the internal  $W$ -world, thus, they cannot be discovered in experiments nor at any energy range,*

are the three solid implications of the resulting M\$MSM for the experiments at LEP2, at the Tevatron and at LHC, which are drastically different from those of MSSM models. Which of these schemes, if any, is realized either exactly or at least approximately in nature remains to be seen in the years to come.

## 2 Preliminaries

To facilitate the physical picture and provide sufficient background, this section contains some of the necessary preliminaries on generic of the OMM formalism, which one to know in order to understand a structure of suggested SUSY approach. Since it is too technical for present article, we outline only relevant steps in concise schematic form hoping to supplement this shortage of insufficient rigorous treatment by referring to [1,2] for more detailed justification of some of the procedure and complete exposition. The present article is a direct continuation of [1,2], so we adopt its all ideas and conventions. In the next section and further we shall deal with the MW geometry, except for the change of the concept of quark inserted schematically here to subquark defined on the given internal world. To be brief we often suppress the indices without notice.

We start by tracing at elementary level the relevant steps of motivation of the OM for-

malism:

- First step is an extension of the Minkowski space  $M_4 \rightarrow M_8 = M_4 \oplus M_4$  in order to introduce the particle mass operator defined on the internal wold  $M_4$  of the inner degrees of freedom. For example, in a case of Dirac's particle one proceeds at once:

$$\left( \underbrace{\gamma p}_x - m \right) \psi_x = 0 \quad \rightarrow \quad \gamma p \psi = 0,$$

provided by  $\psi = \psi_x \psi_u$ ,  $\gamma p = \underbrace{\gamma p}_x - \underbrace{\gamma p}_u$ ,  $m \psi_u \equiv \underbrace{\gamma p}_u \psi_u$  and

$$d x^2 = inv \quad \rightarrow \quad d x_8^2 = d x^2 - d u^2 = 0, \quad x_8 \in M_8.$$

The same holds for the other fields of arbitrary spin.

- Next, a two-steps passage  $M_4 \rightarrow M_6 \xrightarrow{45^0} G_6$  will be performed for each sample of the  $M_4$ .

a) A passage  $M_4 \rightarrow M_6$  restores the complete equivalence between the three spacial and three time components:

$$e_4 = (\vec{e}, e_0) \quad \rightarrow \quad \vec{e}_6 = (\vec{e}, \vec{e}_0) \in M_6, \quad x_4 = (\vec{x}, x_0) \quad \rightarrow \quad x_6 = (\vec{x}, \vec{x}_0) \in M_6.$$

b) A rotation  $M_6 \xrightarrow{45^0} G_6$  of the basis vectors on the angle  $45^0$  provides an adequate algebra for quantization of the geometry (subsec.2.1):

$$\begin{aligned} \vec{e}_6 &\xrightarrow{45^0} e_{(\lambda\alpha)}, \quad \lambda = \pm, \quad \alpha = 1, 2, 3, \\ e_{\pm\alpha} &= \frac{1}{\sqrt{2}}(e_{0\alpha} \pm e_\alpha) = O_\pm \otimes \sigma_\alpha, \quad < O_\lambda, O_\tau > = 1 - \delta_{\lambda\tau}, \quad < \sigma_\alpha, \sigma_\beta > = \delta_{\alpha\beta}. \end{aligned}$$

Accordingly one gets  $M_8 \rightarrow G_{12}$ . Thus, within a simplified scheme (one  $u$ -channel) of the following it is convenient to deal in terms of smooth differentiable manifold

$$G = G_\eta \oplus G_u,$$

$Dim G = 12$ ,  $Dim G_i = 6$  ( $i = \eta, u$ ). Note that presumably we are allowed to perceive directly only the  $G_\eta$  which will be related to the spacetime continuum, but not the  $G_u$  which will be displayed as a space of inner degrees of freedom (see below).

- Finally, in suggested approach we will be dealing in terms of first degree of the line element, which entails an additional phase multiplier  $\Phi(\zeta)$  for the vector defined on  $G$ :

$$d \zeta^2 \quad \rightarrow \quad d \vec{\zeta} e^{iS}, \quad \vec{\zeta} \quad \rightarrow \quad \vec{\Phi}(\zeta) = \vec{\zeta} \Phi(\zeta), \quad \Phi(\zeta) \equiv e^{iS},$$

where  $\vec{\zeta} = \vec{e} \zeta$ ,  $\vec{e} = (\vec{e}_\eta, \vec{e}_u)$ ,  $S(\zeta)$  is the invariant action defined on  $G$ .

The  $\{e_{(\lambda,\mu,\alpha)} = O_{\lambda,\mu} \otimes \sigma_\alpha\} \subset G$  ( $\lambda, \mu = 1, 2$ ;  $\alpha = 1, 2, 3$ ) are linear independent 12 unit vectors at the point  $p$  of the manifold  $G$ , provided by the linear unit bipseudovectors  $O_{\lambda,\mu}$  implying

$$< O_{\lambda,\mu}, O_{\tau,\nu} > = {}^* \delta_{\lambda,\tau} {}^* \delta_{\mu,\nu} \quad < \sigma_\alpha, \sigma_\beta > = \delta_{\alpha\beta}, \quad {}^* \delta = 1 - \delta,$$

where  $\delta$  is Kronecker symbol, the  $\{O_{\lambda,\mu} = O_\lambda \otimes O_\mu\}$  is the basis for tangent vectors of  $2 \times 2$  dimensional linear pseudospace  ${}^* \mathbf{R}^4 = {}^* \mathbf{R}^2 \otimes {}^* \mathbf{R}^2$ , the  $\sigma_\alpha$  refers to three dimensional

ordinary space  $\mathbf{R}^3$ . Henceforth we always let the first two subscripts in the parentheses to denote the pseudovector components, while the third refers to the ordinary vector components. The metric on  $G$  is  $\hat{\mathbf{g}} : \mathbf{T}_p \otimes \mathbf{T}_p \rightarrow C^\infty(G)$  a section of conjugate vector bundle  $S^2\mathbf{T}$ . Any vector  $\mathbf{A}_p \in \mathbf{T}_p$  reads  $\mathbf{A} = eA$ , provided with components  $A$  in the basis  $\{e\}$ . In holonomic coordinate basis  $(\partial/\partial\zeta)_p$  one gets  $A = \left. \frac{d\zeta}{dt} \right|_p$  and  $\hat{g} = g d\zeta \otimes d\zeta$ .

The manifold  $G$  decomposes as follows:

$$G = {}^*\mathbf{R}^2 \otimes {}^*\mathbf{R}^2 \otimes \mathbf{R}^3 = G_\eta \oplus G_u = \sum_{\lambda, \mu=1}^2 \oplus \mathbf{R}_{\lambda\mu}^3 = \mathbf{R}_x^3 \oplus \mathbf{R}_{x_0}^3 \oplus \mathbf{R}_u^3 \oplus \mathbf{R}_{u_0}^3$$

employing corresponding basis vectors  $e_{i(\lambda\alpha)} = O_\lambda \otimes \sigma_\alpha \subset G_i$  ( $\lambda = \pm$ ,  $i = \eta, u$ ) of tangent sections, where

$$O_{i+} = \frac{1}{\sqrt{2}}(O_{1,1} + \varepsilon_i O_{2,1}), \quad O_{i-} = \frac{1}{\sqrt{2}}(O_{1,2} + \varepsilon_i O_{2,2}), \quad \varepsilon_\eta = 1, \quad \varepsilon_u = -1,$$

and  $\langle O_{i\lambda}, O_{j\tau} \rangle = \varepsilon_i \delta_{ij}^* \delta_{\lambda\tau}$ . The  $G_\eta$  decomposes into three dimensional ordinary and time flat spaces

$$G_\eta = \mathbf{R}_x^3 \oplus \mathbf{R}_{x_0}^3$$

with signatures  $sgn(\mathbf{R}_x^3) = (+++)$  and  $sgn(\mathbf{R}_{x_0}^3) = (---)$ . The same holds for  $G_u$  with the opposite signatures  $sgn(\mathbf{R}_u^3) = (---)$  and  $sgn(\mathbf{R}_{u_0}^3) = (+++)$ .

The passage to Minkowski space is a further step as follows: Since all directions in  $\mathbf{R}_{x_0}^3$  are equivalent, then by notion *time* one implies the projection of time-coordinate on fixed arbitrary universal direction in  $\mathbf{R}_{x_0}^3$ . This clearly respects the physical ground. By the reduction  $\mathbf{R}_{x_0}^3 \rightarrow \mathbf{R}_{x_0}^1$  the passage

$$G_\eta \rightarrow M_4 = \mathbf{R}_x^3 \oplus \mathbf{R}_{x_0}^1$$

may be performed whenever it will be necessary.

In the case of gravity, the passage from six dimensional curved manifold  $\tilde{G}$  to four dimensional Riemannian geometry  $R_4$  is straightforward by making use of reduction of three time components  $e_{0\alpha} = \frac{1}{\sqrt{2}}(e_{(+\alpha)} + e_{(-\alpha)})$  of basis sixvector  $e_{(\lambda\alpha)}$  to the single one  $e_0$  in the given universal direction, which merely fixed a time coordinate. Actually, since Lagrangian of the fields defined on  $\tilde{G}$  is a function of scalars such as  $A_{(\lambda\alpha)} B^{(\lambda\alpha)} = A_{0\alpha} B^{0\alpha} + A_\alpha B^\alpha$ , thus taking into account that  $A_{0\alpha} B^{0\alpha} = A_{0\alpha} \langle e^{0\alpha}, e^{0\beta} \rangle B_{0\beta} = A_0 \langle e^0, e^0 \rangle B_0 = A_0 B^0$ , one readily may perform the required passage. In this case one has

$$d\zeta^2 = d\eta^2 - du^2 = 0, \quad d\eta^2|_{6 \rightarrow 4} \equiv ds^2 = g_{\mu\nu} dx^\mu dx^\nu = du^2 = inv.$$

For more discussion see [1].

## 2.1 Quantization of geometry

We proceed at once with the secondary quantization of geometry by substituting the pseudo vectors  $O_\lambda$  for the operators supplied by additional index  $(r)$  referring to the quantum numbers of corresponding state

$$\hat{O}_1^r = O_1^r \alpha_1, \quad \hat{O}_2^r = O_2^r \alpha_2, \quad \{\hat{O}_\lambda^r, \hat{O}_\tau^{r'}\} = \delta_{rr'} {}^* \delta_{\lambda\tau} I_2,$$

where  $\{\alpha_\lambda, \alpha_\tau\} = {}^* \delta_{\lambda\tau} I_2$ ,  $\alpha^\lambda = {}^* \delta^{\lambda\mu} \alpha_\mu = (\alpha_\lambda)^+$ , For example  $\alpha_1 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$ . Then

$$\hat{O}_1^r |0\rangle = O_1^r |1\rangle, \quad \hat{O}_2^r |1\rangle = O_2^r |0\rangle.$$

Hence  $\hat{O}_1^r |1\rangle = 0$ ,  $\hat{O}_2^r |0\rangle = 0$ . A matrix realization of the states is

$$|0\rangle \equiv \chi_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad |1\rangle \equiv \chi_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

Also, instead of ordinary basis vectors we introduce the operators  $\hat{\sigma}_\alpha^r \equiv \delta_{\alpha\beta\gamma} \sigma_\beta^r \tilde{\sigma}_\gamma$ , where  $\tilde{\sigma}_\gamma$  are Pauli's matrices, and

$$\langle \sigma_\alpha^r, \sigma_\beta^{r'} \rangle = \delta_{rr'} \delta_{\alpha\beta}, \quad \hat{\sigma}_r^\alpha = \delta^{\alpha\beta} \hat{\sigma}_\beta^r = (\hat{\sigma}_\alpha^r)^+ = \hat{\sigma}_\alpha^r, \quad \{\hat{\sigma}_\alpha^r, \hat{\sigma}_\beta^{r'}\} = 2\delta_{rr'} \delta_{\alpha\beta} I_2.$$

Than, the states  $|0\rangle \equiv \varphi_{1(\alpha)}$  and  $|1\rangle \equiv \varphi_{2(\alpha)}$  are as follows:

$$\varphi_{1(\alpha)} \equiv \chi_1, \quad \varphi_{2(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \varphi_{2(2)} = \begin{pmatrix} -i \\ 0 \end{pmatrix}, \quad \varphi_{2(3)} = \begin{pmatrix} 0 \\ -1 \end{pmatrix},$$

and

$$\hat{\sigma}_\alpha^r \varphi_{1(\alpha)} = \sigma_\alpha^r \varphi_{2(\alpha)} = (\sigma_\alpha^r \tilde{\sigma}_\alpha) \varphi_{1(\alpha)}, \quad \hat{\sigma}_\alpha^r \varphi_{2(\alpha)} = \sigma_\alpha^r \varphi_{1(\alpha)} = (\sigma_\alpha^r \tilde{\sigma}_\alpha) \varphi_{2(\alpha)}.$$

Hence, the single eigenvalue  $(\sigma_\alpha^r \tilde{\sigma}_\alpha)$  associates with different  $\varphi_{\lambda(\alpha)}$ , namely it is degenerated with degeneracy degree equal 2. Thus, among quantum numbers  $r$  there is also the quantum number of the half integer spin  $\vec{\sigma}$  ( $\sigma_3 = \frac{1}{2}s$ ,  $s = \pm 1$ ). This consequently gives rise to the spins of particles. Next we introduce the operator

$$\hat{\gamma}_{(\lambda,\mu,\alpha)}^r \equiv \hat{O}_\lambda^{r_1} \otimes \hat{O}_\mu^{r_2} \otimes \hat{\sigma}_\alpha^{r_3}$$

and the state vector

$$\chi_{\lambda,\mu,\tau(\alpha)} \equiv |\lambda, \mu, \tau(\alpha)\rangle = \chi_\lambda \otimes \chi_\mu \otimes \varphi_{\tau(\alpha)},$$

where  $\lambda, \mu, \tau, \nu = 1, 2$ ;  $\alpha, \beta = 1, 2, 3$  and  $r \equiv (r_1, r_2, r_3)$ . Omitting two valuedness of state vector we apply  $|\lambda, \tau, \delta(\beta)\rangle \equiv |\lambda, \tau\rangle$ , and remember that always the summation must be extended over the double degeneracy of the spin states ( $s = \pm 1$ ). The matrix elements read

$$\langle \lambda, \mu | \hat{\gamma}_{(\tau,\nu,\alpha)}^r | \tau, \nu \rangle = {}^* \delta_{\lambda\tau} {}^* \delta_{\mu\nu} e_{(\tau,\nu,\alpha)}^r, \quad \langle \tau, \nu | \hat{\gamma}_r^{(\tau,\nu,\alpha)} | \lambda, \mu \rangle = {}^* \delta_{\lambda\tau} {}^* \delta_{\mu\nu} e_r^{(\tau,\nu,\alpha)}.$$

for given  $\lambda, \mu$ . The operators of occupation numbers are

$$\hat{N}_{\alpha\beta}^{rr'} = \hat{\gamma}_{(1,1,\alpha)}^r \hat{\gamma}_{(2,2,\beta)}^{r'}, \quad \hat{N}_{\alpha\beta}^{rr'} = \hat{\gamma}_{(2,1,\alpha)}^r \hat{\gamma}_{(1,2,\beta)}^{r'},$$



with the expectation values implying Pauli's exclusion principle. The set of operators  $\{\hat{\gamma}^r\}$  is the basis for tangent operator vectors  $\hat{\Phi}(\zeta) = \hat{\gamma}^r \Phi_r(\zeta)$  of the 12 dimensional flat operator manifold  $\hat{G}$ , where we introduce the vector function of ordinary class of functions of  $C^\infty$  smoothness defined on the manifold  $G$ :  $\Phi_r^{(\lambda, \mu, \alpha)}(\zeta) = \zeta^{(\lambda, \mu, \alpha)} \Phi_r^{\lambda, \mu}(\zeta)$ ,  $\zeta \in G$ .

The basis  $\{\hat{\gamma}^r\}$  decomposes ( $\lambda = \pm$ ;  $\alpha = 1, 2, 3$ ;  $i = \eta, u$ ):

$$\hat{\gamma}_{(+\alpha)}^r = \frac{1}{\sqrt{2}}(\gamma_{(1,1\alpha)}^r + \varepsilon_i \gamma_{(2,1\alpha)}^r), \quad \hat{\gamma}_{(-\alpha)}^r = \frac{1}{\sqrt{2}}(\gamma_{(1,2\alpha)}^r + \varepsilon_i \gamma_{(2,2\alpha)}^r).$$

The expansions of operator vectors  $\hat{\Psi}_i \in \hat{G}$  and operator covectors  $\bar{\hat{\Psi}}_i$  are written  $\hat{\Psi}_i = \hat{\gamma}_i^r \psi_r$ ,  $\bar{\hat{\Psi}}_i = \hat{\gamma}_i^r \psi_r$ . As to the vector functions, they are defined on the manifold  $G$ :

$$\psi_r^{(\pm\alpha)}(\eta, p_\eta) = \eta^{(\pm\alpha)} \psi_r^\pm(\eta, p_\eta), \quad \psi_r^{(\pm\alpha)}(u, p_u) = u^{(\pm\alpha)} \psi_r^\pm(u, p_u).$$

Due to the spin states, the  $\psi_r^\pm$  is the Fermi field of the positive and negative frequencies  $\psi_r^\pm = \psi_{\pm p}^r$ .

## 2.2 Realization of the flat manifold $G$

The bispinor  $\Psi(\zeta)$  defined on manifold  $G = G_\eta \oplus G_u$  is written

$$\Psi(\zeta) = \psi_\eta(\eta) \psi_u(u),$$

where the  $\psi_i$  is a bispinor defined on the manifold  $G$ . The free state of  $i$ -type fermion with definite values of momentum  $p_i$  and spin projection  $s$  is described by means of plane waves. We make use of localized wave packets of operator vector fields  $\hat{\Psi}_i$  and  $\hat{\Phi}(\zeta)$ . In this manner we get the important relation

$$\begin{aligned} \sum_{\lambda=\pm} < \chi_\lambda | \hat{\Phi}(\zeta) \bar{\hat{\Phi}}(\zeta) | \chi_\lambda > &= \sum_{\lambda=\pm} < \chi_\lambda | \bar{\hat{\Phi}}(\zeta) \hat{\Phi}(\zeta) | \chi_\lambda > = \\ &= -i \zeta^2 G_\zeta(0) = -i \left( \eta^2 G_\eta(0) - u^2 G_u(0) \right), \end{aligned}$$

where  $G_i(0) \equiv \lim_{i \rightarrow i'} G_i(i - i')$ , ( $i = \zeta, \eta, u$ ), etc., the Green's function  $G_i(i - i') = -(i \hat{\partial}_i + m) \Delta_i(i - i')$  is provided by the usual invariant singular functions  $\Delta_i(i - i')$  ( $i = \eta, u$ ), the state vectors  $\chi_\lambda$  are given in App. A. Realization of the manifold  $G$  ensued from the constraint imposed upon the matrix element of bilinear form, which is, as a geometric object, required to be finite

$$\sum_{\lambda=\pm} < \chi_\lambda | \hat{\Phi}(\zeta) \bar{\hat{\Phi}}(\zeta) | \chi_\lambda > < \infty,$$

which gives rise to

$$\zeta^2 G_F(0) < \infty,$$

and

$$G_F(0) = G_F(0) = G_F(0) =$$

$$= \lim_{u \rightarrow u'} \left[ -i \sum_{\vec{p}_u} \psi_{p_u}(u) \bar{\Psi}_{p_u}(u') \theta(u_0 - u'_0) + i \sum_{\vec{p}_u} \bar{\Psi}_{p_u}(u') \psi_{p_u}(u) \theta(u'_0 - u_0) \right],$$

where  $G_F$ ,  $G_F$  and  $G_F$  are the causal Green's functions characterized by the boundary condition that only positive frequency occur for  $\eta_0 > 0$  ( $u_0 > 0$ ), only negative for  $\eta_0 < 0$  ( $u_0 < 0$ ). Here  $\eta_0 = |\vec{\eta}_0|$ ,  $\eta_{0\alpha} = \frac{1}{\sqrt{2}}(\eta_{(+\alpha)} + \eta_{(-\alpha)})$  and the same holds for  $u_0$ . Then, satisfying the condition eq.(2.2.3) a length of each vector  $\zeta = e\zeta \in G$  compulsory should be equaled zero

$$\zeta^2 = \eta^2 - u^2 = 0.$$

Relativity principle holds

$$d\eta^2 \Big|_{6 \rightarrow 4} \equiv ds^2 = du^2 = inv.$$

## 2.3 Primordial structures and link establishing processes

In [1] we have chosen a simple setting and considered the primordial structures designed to possess certain physical properties satisfying the stated general rules. These structures are thought to be the substance out of which the geometry and particles are made. We distinguish  $\eta$ - and  $u$ -types primordial structures involved in the linkage establishing processes occurring between the structures of different types.

The  $\eta$ -type structure may accept the linkage only from  $u$ -type structure, which is described by the link function  $\psi(\eta)$  belonging to the ordinary class of functions of  $C^\infty$  smoothness, where  $\eta = e_{(\lambda\alpha)} \eta^{(\lambda\alpha)}$ , ( $\lambda = \pm; \alpha = 1, 2, 3$ , see subsec.2.1),  $\eta$  is the link coordinate. Respectively the  $u$ -type structure may accept the linkage only from  $\eta$ -type structure described by the link function  $\psi(u)$  ( $u$ -channel,  $u = e_u u$ ), where

$$\psi^{(\pm\alpha)}_{\eta}(\eta, p_{\eta}) = \eta^{(\pm\alpha)} \psi^{\pm}_{\eta}(\eta, p_{\eta}), \quad \psi^{(\pm\alpha)}_u(u, p_u) = u^{(\pm\alpha)} \psi^{\pm}_u(u, p_u),$$

a bispinor  $\psi^{\pm}_i$  is the invariant state wave function of positive or negative frequencies,  $p_i$  is the corresponding link momentum. Thus, a primordial structure can be considered as a fermion. A simplest system made of two structures of different types becomes stable only due to the stable linkage

$$\left| p \right|_{\eta} = (p^{(\lambda\alpha)}_{\eta}, p_{\eta}^{(\lambda\alpha)})^{1/2} = \left| p \right|_u = (p^{(\lambda\alpha)}_u, p_u^{(\lambda\alpha)})^{1/2}.$$

Otherwise they are unstable. There is not any restriction on the number of primordial structures of both types involved in the link establishing processes simultaneously. Only, in the stable system the link stability condition must be held for each linkage separately. Suppose that persistent processes of creation and annihilation of the primordial structures proceed in different states  $s, s', s'', \dots$ . The "creation" of structure in the given state ( $s$ ) is due to its transition to this state from other states ( $s', s'', \dots$ ), while the "annihilation" means a vice versa. Satisfying the stability condition the primordial structures from

arbitrary states can establish a stable linkage. Among the states  $(s, s', s'', \dots)$  there is a lowest one  $(s_0)$ , in which all structures are regular, i.e., they are in free (pure) state and described by the plane wave functions  $\psi_{\eta}^{\pm}(\eta_f, p_{\eta})$  or  $\psi_u^{\pm}(u_f, p_u)$  defined respectively on flat manifolds  $G_{\eta}$  and  $G_u$ . The index (f) specifies the points of corresponding flat manifolds  $\eta_f \in G_{\eta}$ ,  $u_f \in G_u$ . Note that the processes of creation and annihilation of regular structures in lowest state are described by the OM formalism given above.

## 2.4 Distorted primordial structures

In all higher states the primordial structures are distorted (interaction states) and described by distorted link functions defined on distorted manifolds  $\tilde{G}_{\eta}$  and  $\tilde{G}_u$ . The distortion  $G \rightarrow \tilde{G}$  with hidden Abelian local group  $G = U^{loc}(1) = SO^{loc}(2)$  and one dimensional trivial algebra  $\hat{g} = R^1$  is considered in [63]. Within that scheme the basis  $e^f$  undergoes distortion transformation  $e(\theta) = D(\theta) e^f$ . The matrix  $D(\theta)$  is in the form  $D(\theta) = C \otimes R(\theta)$ , where  $O_{(\lambda\alpha)} = C_{(\lambda\alpha)}^{\tau} O_{\tau}$  and  $\sigma_{(\lambda\alpha)}(\theta) = R_{(\lambda\alpha)}^{\beta}(\theta) \sigma_{\beta}$ . Here  $R(\theta)$  is the matrix of the group  $SO(3)$  of ordinary rotations of the planes involving two arbitrary basis vectors of the spaces  $R_{\pm}^3$  around the orthogonal third axes  $(\pm k)$  through the angle  $(\theta_{\pm k})$ . The relation between the wave functions of distorted and regular structures reads

$$\psi_u^{\lambda}(\theta_{+k}) = f_{(+)}(\theta_{+k}) \psi_u^{\lambda}, \quad \psi_u^{\lambda}(\theta_{-k}) = \psi_u^{\lambda} f_{(-)}(\theta_{-k}),$$

where  $\psi_u^{\lambda}$  ( $\psi_{\lambda}$ ) is the plane wave function of regular ordinary structure (antistructure). Next, we supplement the previous assumptions given in subsec.2.3 by the new one that now the  $\eta$ -type (fundamental) regular structure can not directly form a stable system with the regular  $u$ -type (ordinary) structures. Instead of it the  $\eta$ -type regular structure forms a stable system with the infinite number of distorted ordinary structures, where the link stability condition held for each linkage separately. Such structures take part in realization of flat manifold  $G$ . We employ the wave packets constructed by superposition of these functions furnished by generalized operators of creation and annihilation as the expansion coefficients. Geometry realization condition now should be satisfied for each ordinary structure in terms of

$$G_u^{\theta}(0) = \lim_{\theta_+ \rightarrow \theta_-} G_u^{\theta}(\theta_+ - \theta_-) = G_{\eta}^{\theta}(0) = \lim_{\eta'_f \rightarrow \eta_f} G_{\eta}^{\theta}(\eta'_f - \eta_f).$$

Then

$$\sum_k \psi_u(\theta_{+k}) \bar{\psi}_u(\theta_{-k}) = \sum_k \psi'_u(\theta'_{+k}) \bar{\psi}'_u(\theta'_{-k}) = \dots = inv.$$

Namely, the distorted ordinary structures emerge in geometry only in permissible combinations forming a stable system. Below, in simplified schematic way we exploit the background of the known colour confinement and gauge principles. Naive version of such construction still should be considered as a preliminary one, which will be further elaborated to make sense in the sec.3.

## 2.5 Quarks and colour confinement

At the very first to avoid irrelevant complications, here, for illustrative purposes, we will attempt to introduce temporarily skeletonized “quark” and “antiquark” fields emerged in

confined phase in the simplified geometry with the one- $u$  channel given in the previous subsections. The complete picture of such a dynamics is beyond the scope of this subsection, but some relevant discussions on this subject will also be presented in the subsec.3.2. We may think of the function  $\psi_u^\lambda(\theta_{+k})$  at fixed  $(k)$  as being  $u$ -component of bispinor field of "quark"  $q_k$ , and of  $\bar{\psi}_u^\lambda(\theta_{-k})$  - an  $u$ -component of conjugated bispinor field of "antiquark"  $\bar{q}_k$ . The index  $(k)$  refers to colour degree of freedom in the case of rotations through the angles  $\theta_{+k}$  and anticolour degree of freedom in the case of  $\theta_{-k}$ . The  $\eta$ -components of quark fields are plane waves. There are exactly three colours. The rotation through the angle  $\theta_{+k}$  yields a total quark field defined on the flat manifold  $G = G_\eta \oplus G_u$

$$q_k(\theta) = \Psi(\theta_{+k}) = \psi_\eta^0 \psi_u(\theta_{+k})$$

where  $\psi_\eta^0$  is a plane wave defined on  $G_\eta$ . This allows an other interpretation of quarks, which is absolutely equivalent to the former one and will be widely used throughout this article, i.e.,

$$q_k(\theta) = \psi_\eta^0 q_k(\theta) = q_k(\theta) \psi_u^0, \quad q_k(\theta) \equiv f_{(+)}(\theta_{+k}) \psi_\eta^0,$$

where  $\psi_u^0$  is a plane wave,  $q_k(\theta)$  and  $q_k(\theta)$  may be considered as the quark fields with the same quantum numbers defined respectively on flat manifolds  $G_u$  and  $G_\eta$ . Making use of the rules stated one may readily return to Minkowski space  $G_\eta \rightarrow M_4$ . In the sequel, a conventional quark fields defined on  $M_4$  will be ensued  $q_k(\theta) \rightarrow q_k(x)$ ,  $x \in M_4$ . They imply

$$\sum_k q_{kp} \bar{q}_{kp} = \sum_k q'_{kp} \bar{q}'_{kp} = \dots = inv.$$

It utilizes the idea of colour confinement principle: the quarks emerge in the geometry only in special combinations of colour singlets. Only two colour singlets are available (see below)

$$(q\bar{q}) = \frac{1}{\sqrt{3}} \delta_{kk'} \hat{q}_k \bar{\hat{q}}_{k'} = inv, \quad (qqq) = \frac{1}{\sqrt{6}} \varepsilon_{klm} \hat{q}_k \hat{q}_l \hat{q}_m = inv.$$

## 2.6 Gauge principle-internal symmetries

Each regular structure in the lowest state can be regarded as a result of transition from an arbitrary state, in which they assumed to be distorted. Hence, the following transformations may be implemented upon distorted ordinary structures

$$\psi_u^{\lambda'}(\theta'_{+l}) = f_{lk}^{(+)} \psi_u^\lambda(\theta_{+k}) = f(\theta'_{+l}, \theta_{-k}) \psi_u^\lambda(\theta_{+k}), \quad f(\theta'_{+l}, \theta_{-k}) = f_{(+)}(\theta'_l) f_{(-)}(\theta_k). \quad (2.6.1)$$

The transformation functions are the operators in the space of internal degrees of freedom labeled by  $(\pm k)$  corresponding to distortion rotations around the axes  $(\pm k)$  by the angles  $\theta_{\pm k}$ . We make proposition that the distortion rotations are incompatible, namely the transformation operators  $f_{lk}^{(\pm)}$  obey the incompatibility relations

$$\begin{aligned} f_{lk}^{(+)} f_{cd}^{(+)} - f_{ld}^{(+)} f_{ck}^{(+)} &= \|f^{(+)}\| \varepsilon_{lcm} \varepsilon_{kdn} f_{nm}^{(-)}, \\ f_{kl}^{(-)} f_{dc}^{(-)} - f_{dl}^{(-)} f_{kc}^{(-)} &= \|f^{(-)}\| \varepsilon_{lcm} \varepsilon_{kdn} f_{mn}^{(+)}, \end{aligned} \quad (2.6.2)$$

where  $l, k, c, d, m, n = 1, 2, 3$ . This relations hold in general for both local and global rotations. Then one gets the transformations implemented upon the quark field, which in matrix notation take the form  $q'(\zeta) = U(\theta(\zeta))q(\zeta)$ ,  $\bar{q}'(\zeta) = \bar{q}(\zeta)U^+(\theta(\zeta))$ , where  $q = \{q_k\}$ ,  $U(\theta) = \{f_{lk}^{(+)}\}$ . Due to the incompatibility commutation relations the transformation matrices  $\{U\}$  generate the unitary group of internal symmetries  $U(1), SU(2), SU(3)$ , while an action of physical system must be invariant under such transformations (Gauge principle).

### 3 Operator multimanifold $\hat{G}_N$

The OM formalism  $\hat{G} = \hat{G}_\eta \oplus \hat{G}_u$  is built up by assuming an existence only of ordinary primordial structures of one sort (one u-channel). To develop the microscopic approach to field theory based on MW geometry, henceforth instead of one sort of ordinary structures we are going to deal with different species of ordinary structures. That is, before we enlarge the previous model we must make an additional assumption concerning an existence of infinite number of  $^i u$ -type ordinary structures of different species  $i = 1, 2, \dots, N$  (multi-u channel). These structures will be specified by the superscript to the left. This hypothesis, as it will be seen in the following, leads to the progress of understanding of the properties of particles. At the very outset we consider the processes of creation and annihilation of regular structures of  $\eta$ - and  $^i u$ -types in the lowest state ( $s_0$ ). The general rules stated in subsec 2.1 regarding to this change apply a substitution of operator basis pseudo vectors and covectors by a new ones supplied with additional superscript ( $i$ ) to the left referred to different species ( $i = 1, 2, \dots, N$ )

$$^i \hat{O}_{\lambda, \mu}^{r_1 r_2} = ^i \hat{O}_\lambda^{r_1} \otimes ^i \hat{O}_\mu^{r_2} \equiv ^i \hat{O}_{\lambda, \mu}^r = ^i O_{\lambda, \mu}^r (\alpha_\lambda \otimes \alpha_\mu), \quad (3.1)$$

provided  $r = (r_1, r_2)$  and

$$\begin{aligned} ^i O_{1,1}^r &= \frac{1}{\sqrt{2}}(\nu_i O_{\eta+}^r + ^i O_{u+}^r), & ^i O_{2,1}^r &= \frac{1}{\sqrt{2}}(\nu_i O_{\eta+}^r - ^i O_{u+}^r), \\ ^i O_{1,2}^r &= \frac{1}{\sqrt{2}}(\nu_i O_{\eta-}^r + ^i O_{u-}^r), & ^i O_{2,2}^r &= \frac{1}{\sqrt{2}}(\nu_i O_{\eta-}^r - ^i O_{u-}^r), \end{aligned}$$

where

$$< \nu_i, \nu_j > = \delta_{ij}, \quad < ^i O_{u\lambda}^r, ^i O_{u\tau}^{r'} > = -\delta_{ij} \delta_{rr'}^* \delta_{\lambda\tau}, \quad < O_{\eta\lambda}^r, ^i O_{u\tau}^{r'} > = 0. \quad (3.2)$$

In analogy with subsec.2.1 we consider the operators  $^i \hat{\gamma}_{(\lambda, \mu, \alpha)}^r = ^i \hat{O}_{\lambda, \mu}^{r_1 r_2} \otimes \hat{\sigma}_\alpha^{r_3}$ . and calculate nonzero matrix elements

$$< \lambda, \mu \mid ^i \hat{\gamma}_{(\tau, \nu, \alpha)}^r \mid \tau, \nu > = {}^* \delta_{\lambda\tau} {}^* \delta_{\mu, \nu} ^i e_{(\tau, \nu, \alpha)}^r, \quad (3.3)$$

where  $^i e_{(\lambda, \mu, \alpha)}^r = ^i O_{\lambda, \mu}^r \otimes \sigma_\alpha$ . The set of operators  $\{^i \hat{\gamma}^r\}$  is the basis for all operator vectors  $\hat{\Phi}(\zeta) = ^i \hat{\gamma}^r {}^i \Phi_r(\zeta)$  of tangent section of principle bundle with the base of operator multimanifold  $\hat{G}_N = \left( \sum_i^N \oplus {}^* \hat{R}_i^4 \right) \otimes \hat{R}^3$ . Here  ${}^* \hat{R}_i^4$  is the  $2 \times 2$  dimensional linear pseudo operator space, with the set of the linear unit operator pseudo vectors eq.(3.1) as the basis of tangent vector section, and  $\hat{R}^3$  is the three dimensional real linear operator space

with the basis consisted of the ordinary unit operator vectors  $\{\hat{\sigma}_\alpha^r\}$ . The  $\hat{G}_N$  decomposes as follows:

$$\hat{G}_N = \hat{G}_\eta \oplus \hat{G}_{u_1} \oplus \cdots \oplus \hat{G}_{u_N}, \quad (3.4)$$

where  $\hat{G}_{u_i}$  is the six dimensional operator manifold of the species ( $i$ ) with the basis

$\left\{ {}^i\hat{\gamma}_{u(\lambda\alpha)}^r = {}^i\hat{O}_\lambda^r \otimes \hat{\sigma}_\alpha^r \right\}$ . The expansions of operator vectors and covectors are written  $\hat{\Psi}_\eta = \hat{\gamma}_\eta^r \psi_r$ ,  $\hat{\Psi}_u = {}^i\hat{\gamma}_u^r {}^i\psi_r$ ,  $\bar{\hat{\Psi}}_\eta = \hat{\gamma}_\eta^r \psi_r$ ,  $\bar{\hat{\Psi}}_u = {}^i\hat{\gamma}_u^r {}^i\psi_r$ , where the components  $\psi_r(\eta)$  and  ${}^i\psi_r(u)$  are respectively the link functions of  $\eta$ -type and  ${}^i u$ -type structures. The quantum field and differential geometric aspects of OMM  $\hat{G}_N$  can be discussed on the analogy of  $\hat{G}_{N=1}$ . Recall that we consider the special system of regular structures made of fundamental structure of  $\eta$ -type and infinite number of  ${}^i u$ -type ordinary structures of different species ( $i = 1, \dots, N$ ). The primordial structures establish a stable linkage to form stable system:

$$p^2 = p_\eta^2 - \sum_{i=1}^N p_{u_i}^2 = 0. \quad (3.5)$$

The state of free ordinary structure of  ${}^i u$ -type with the given values of link momentum  $p_{u_i}$  and spin projection  $s_i$  is described by means of plane wave (see App. A). We also involve the solution of negative frequencies with the normalized bispinor amplitude.

### 3.1 Realization of multimanifold (MM) $G_N$

On analogy of subsec.2.2 we make use of localized wave packets by means of superposition of plane wave solutions furnished by creation and annihilation operators in agreement with Pauli's principle. The new proposal includes and generalizes an early version of the relation

$$\begin{aligned} \sum_{\lambda=\pm} < \chi_\lambda | \hat{\Phi}(\zeta) \bar{\hat{\Phi}}(\zeta) | \chi_\lambda > = \sum_{\lambda=\pm} < \chi_\lambda | \bar{\hat{\Phi}}(\zeta) \hat{\Phi}(\zeta) | \chi_\lambda > = \\ -i \zeta^2 G_\zeta(0) = -i \left( \eta^2 G_\eta(0) - \sum_{i=1}^N u_i^2 G_{u_i}(0) \right). \end{aligned} \quad (3.1.1)$$

provided by the Green's function  $G_{u_i}(u_i - u'_i) = -(i \hat{\partial}_{u_i} + m) \Delta_{u_i}(u_i - u'_i)$ , thus

$$\zeta^2 G_\zeta(0) = \eta^2 G_\eta(0) - \sum_{i=1}^N u_i^2 G_{u_i}(0), \quad (3.1.2)$$

where  $G_\eta$ ,  $G_u$  and  $G_\zeta$  are causal Green's functions of the  $\eta$ -,  $u$ - and  $\zeta$ -type structures. This result is of particular interest because along the same line with sec.2, the realization of the MM stems from the condition imposed upon the matrix element eq.(3.1.1), that as the bilinear form it is required to be finite

$$\sum_{\lambda=\pm} < \chi_\lambda | \hat{\Phi}(\zeta) \bar{\hat{\Phi}}(\zeta) | \chi_\lambda > < \infty, \quad (3.1.3)$$

or

$$\zeta^2 G_\zeta(0) < \infty.$$

Denote  $u^2 G(0) \equiv \lim_{u_i \rightarrow u'_i} \sum_{i=1}^N (u_i u'_i) G_{u_i}(u_i - u'_i)$  and consider a stable system eq.(3.5), i.e., as to the Green's functions they satisfy the condition

$$G_F(0) = G_\eta(0) = G_\zeta(0), \quad (3.1.4)$$

provided  $m \equiv |p_u| = \left( \sum_{i=1}^N p_{u_i}^2 \right)^{1/2} = |p_\eta|$ . According to eq.(3.1.3) and eq.(3.1.4), the length of each vector  $\zeta = {}^i e^i \zeta \in G_N$  should be equaled zero  $\zeta^2 = \eta^2 - u^2 = \eta^2 - \sum_{i=1}^N (u_i^G)^2 = 0$ , where use is made of

$$u_i^G \equiv u_i \left[ \lim_{u_i \rightarrow u'_i} G_F(u_i - u'_i) / \lim_{\eta \rightarrow \eta'} G_F(\eta - \eta') \right]^{1/2},$$

and  $u_i^G = {}^i \hat{e}_{u(\lambda, \alpha)} u_i^{G(\lambda, \alpha)}$ . Thus, the MM  $G_N$  comes into being, which decomposes as follows:

$$G_N = G_\eta \oplus G_{u_1} \oplus \cdots \oplus G_{u_N}. \quad (3.1.5)$$

It brings us to the conclusion: the major requirement eq.(3.1.3) provided by stability condition eq.(3.1) or eq.(3.1.4) yields the flat MM  $G_N$ . Meanwhile the Minkowski flat space  $M_4$  stems from the flat submanifold  $G_\eta$  (subsec.2.1), in which the line element turned out to be invariant. That is, the principle of Relativity comes into being with  $M_4$  ensued from the MW geometry  $G_N$ . In the following we shall use a notion of  $i$ -th internal world for the submanifold  $G_{u_i}$ .

## 3.2 Subquarks and subcolour confinement

Since our discussion within this section in many respects is similar to that of sec.2, here we will be brief. We assume that the distortion rotations through the angles  ${}^i \theta_{+k}$  and  ${}^i \theta_{-k}$   $k = 1, 2, 3$  occur separately in the three dimensional internal spaces  $R_{u_i+}^3$  and  $R_{u_i-}^3$  composing six dimensional distorted submanifold  $G_{u_i} \xrightarrow{\theta} \tilde{G}_{u_i} = R_{u_i+}^3 \oplus R_{u_i-}^3$ . As it is exemplified in subsec.2.4, the laws apply in use the wave packets constructed by superposition of the link functions of distorted ordinary structures furnished by generalized operators of creation and annihilation as the expansion coefficients

$$\hat{\Psi}_u(\theta_+) = \sum_{\pm s} \int \frac{d^3 p_{u_i}}{(2\pi)^{3/2}} \left( {}^i \hat{\gamma}_{(+\alpha)}^k {}^i \psi_u^{(+\alpha)}({}^i \theta_{+k}) + {}^i \hat{\gamma}_{(-\alpha)}^k {}^i \psi_u^{(-\alpha)}({}^i \theta_{+k}) \right), \quad (3.2.1)$$

etc. The fields  ${}^i \psi_u(\theta_{+k})$  and  ${}^i \psi_u(\theta_{-k})$  are defined on the distorted internal spaces  $R_{u_i+}^3$  and  $R_{u_i-}^3$ . The generalized expansion coefficients in eq.(3.2.1) imply

$$\langle \chi_- | \{ {}^i \hat{\gamma}_k^{(+\alpha)}(p_{u_i}, s_i), {}^j \hat{\gamma}_{(+\beta)}^{k'}(p'_{u_j}, s'_j) \} | \chi_- \rangle = -\delta_{ij} \delta_{kk'} \delta_{ss'} \delta_{\alpha\beta} \delta^3(\vec{p}_{u_i} - \vec{p}'_{u_j}). \quad (3.2.2)$$

The condition eq.(3.1.4) of MW geometry realization reduces to

$$\sum_{i=1}^N \omega_i \left[ \lim_{i\theta_+ \rightarrow i\theta_-} G_F^\theta(u_i)(i\theta_+ - i\theta_-) \right] = \lim_{\eta_f \rightarrow \eta'_f} G_\eta(\eta_f - \eta'_f), \quad (3.2.3)$$

provided  $\omega_i = \frac{u_i^2}{u^2}$ . Taking into account the expression of causal Green's function for given ( $i$ ), in the case if

$$\lim_{u_{i_1} \rightarrow u_{i_2}} G_F(u_{i_1} - u_{i_2}) = \lim_{u'_{i_1} \rightarrow u'_{i_2}} G_F(u'_{i_1} - u'_{i_2}) = \dots = inv, \quad (3.2.4)$$

one gets

$$\sum_k {}^i\psi_u(i\theta_{+k}) {}^i\bar{\psi}_u(i\theta_{-k}) = \sum_k {}^i\psi'_u(i\theta'_{+k}) {}^i\bar{\psi}'_u(i\theta'_{-k}) = \dots = inv. \quad (3.2.5)$$

Thus, in the context of MW geometry it is legitimate to substitute a formerly introduced term of "quark" ( $q_k$ ) (sec.2.5) by "subquark" ( ${}^i q_k$ ) defined on the given internal world. Everything said will then remain valid, provided we make also a simple change of colours into subcolours. Hence, we may think of the function  ${}^i\psi_u(i\theta_{+k})$  as being  $u$ -component of bispinor field of subquark ( ${}^i q_k$ ) of species ( $i$ ) with subcolour  $k$ , and respectively  ${}^i\bar{\psi}_u(i\theta_{-k})$ -conjugated bispinor field of antishubcolour ( $k$ ). The subquarks and antishubquarks may be local ( ${}^i q_k$ ) or global ( ${}^i q_k^c$ ). Whence, the subquark ( ${}^i q_k$ ) is the fermion with the half integer spin and subcolour degree of freedom, and, according to eq.(3.2.7), could emerge on mass shell only in confined phase

$$\sum_k {}^i q_{kp} {}^i \bar{q}_{kp} = \sum_k {}^i q'_{kp} {}^i \bar{q}'_{kp} = \dots = inv. \quad (3.2.6)$$

To trace a resemblance with sec.3 in [1], the internal symmetry group  ${}^i G = U(1), SU(2), SU(3)$  enables to introduce the gauge theory in internal world with the subcolour charges as exactly conserved quantities. Thereto the subcolour transformation are implemented on subquark fields right through local and global rotation matrices of group  ${}^i G$  in fundamental representation. Due to the Noether procedure the conservation of global charges ensued from the global gauge invariance of physical system, while reinforced requirement of local gauge invariance may be satisfied as well by introducing the gauge fields with the values in Lie algebra  ${}^i \hat{g}$  of group  ${}^i G$ .

## 4 The subquark algebra and supercharges

According to eq.(2.5.1) the following transformations are implemented upon the subquarks (antishubquarks) on the given (i) internal world:

$$q'_l = f_{lk}^{(+)} q_k, \quad \bar{q}'_l = \bar{q}_k f_{kl}^{(-)}, \quad (4.1)$$

where as well for the next section we left implicit the MW-superscript ( $i$ ) to the left. Then, the following composition rules hold for the transformation functions

$$f_{lk}^{(+)} = f_l \circ f_k^{-1}, \quad \bar{f}_{lk}^{(-)} = \bar{f}_l \circ \bar{f}_k^{-1}, \quad (f_l \circ f_k^{-1})(f_c \circ f_d^{-1}) = (f_l f_c) \circ (f_k^{-1} f_d^{-1}), \quad (4.2)$$



where  $l, k, c, d = 1, 2, 3$ , the transformation functions  $f_k \equiv f_{(+)}(\theta_{+k})$  and  $\bar{f}_l \equiv f_{(-)}(\theta_{-k})$  are the operators in the space of internal degrees of freedom labeled by the subcolour index  $(\pm k)$  such that the rotation through the angle  $\theta_{\pm k}$  yields the subquark (antibusquark) field

$$q_k = f_k q_0, \quad \bar{q}_k = \bar{q}_0 \bar{f}_k. \quad (4.3)$$

The incompatibility commutation relations eq.(2.5.2) with the composition rule eq.(4.2) lead to the following commutation relations

$$[f_l, f_k] = \epsilon_{lkm} \bar{f}_m. \quad (4.4)$$

Whence, the subquarks imply

$$[q_l, q_k] = Q_0 \epsilon_{lkm} \bar{q}_m, \quad Q_0 \equiv q_0^2 / \bar{q}_0. \quad (4.5)$$

Following to [2] the symmetries of the  $C$ - ( $C \equiv s, c, b, t$ ) and  $Q$ - worlds are assumed to be respectively local and global unitary  $diag(SU(3))$  symmetries (see subsec.12.1 and 12.2 ), for which the eq.(4.5) reduced to

$$q_l q_k = Q'_0 \epsilon_{lkm} \bar{q}_m, \quad (4.6)$$

while for the  $W$ -world with the unified symmetry  $SU(2)_L \otimes U(1)$  (subsec.12.8) the eq.(4.5) reduced to

$$[q_{1L}, q_{2L}] = Q_0 \bar{q}_{2R}, \quad [q_{2L}, q_{2R}] = Q_0 \bar{q}_{1L}, \quad [q_{2R}, q_{1L}] = Q_0 \bar{q}_{2L}, \quad (4.7)$$

where the subcolour singlets are  $Q_{2R}, [q_{1L}, q_{2L}]$  and  $(q \bar{q})$ . Hence, for the electron and corresponding neutrino one gets (subsec.12.4)

$$[\nu_L, e_L] = Q_0 \bar{e}_R, \quad [e_L, e_R] = Q_0 \bar{\nu}_L, \quad [e_R, \nu_L] = Q_0 \bar{e}_L. \quad (4.8)$$

The eq.(4.5) yields the important relation between the fermionic ( $F$ ) and bosonic ( $B$ ) subcolour singlets

$$(qqq) \equiv \frac{1}{\sqrt{6}} \epsilon_{lkm} q_l q_k q_m = Q_u (q \bar{q}) \equiv Q_u \frac{1}{\sqrt{3}} (q_k \bar{q}_k), \quad F \rightarrow Q_u B, \quad (4.9)$$

and vice versa, where  $F \equiv (qqq)$ ,  $B \equiv (q \bar{q})$ ,  $Q_u \equiv \frac{1}{\sqrt{2}} Q_0$ . It means that considered physical system must respect the invariance under a symmetry group of the fermion-boson transformations occurred in the internal worlds. The latter is known as a “supersymmetry”. It is why the basis vectors in the Hilbert space  $\mathcal{H}$  have taken to be in the form  $|n_B n_F\rangle$ , where the boson and fermion occupation numbers respectively are  $n_B = 1, 2, \dots, \infty$  and  $n_F = 0, 1$ . It is convenient, then, to describe such a quantum me-

chanical system as the spin-1/2 like supersymmetric particle with mass  $m = \left(\frac{\hbar}{Q_u}\right)^2$  moving along the one-dimensional Euclidean line  $\mathcal{R}$ . Therefore, one introduces a generalized bosonic operator  $b$  and a fermionic operator  $f$  acting on the Hilbert space  $\mathcal{H} = L^2(\mathcal{R}) \otimes \mathcal{C}^2$ :

$$\begin{aligned} b : L^2(\mathcal{R}) &\rightarrow L^2(\mathcal{R}), \quad b = \frac{1}{2} \left( \frac{\partial}{\partial u} + W(u) \right) \\ f : \mathcal{C}^2 &\rightarrow \mathcal{C}^2, \quad f = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \end{aligned} \quad (4.10)$$

where the supersymmetric potential  $W(u) : \mathcal{R} \rightarrow \mathcal{R}$  defined on the given (i) internal world is assumed to be piecewise continuously differentiable function. The commutation and anticommutation relations for these operators read

$$[b, b^+] = W'(u), \quad \{f, f^+\} = 1. \quad (4.11)$$

Employing standard technique, in a line with eq.(4.9), next we define the nilpotent supercharge operators

$$Q_u = Q_0 b \otimes f^+ = Q_0 \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix}, \quad Q_u^+ = Q_0 b^+ \otimes f = Q_0 \begin{pmatrix} 0 & 0 \\ b^+ & 0 \end{pmatrix}, \quad (4.12)$$

which obey the anticommutation relations

$$\{Q_u, Q_u\} = \{Q_u^+, Q_u^+\} = 0, \quad (4.13)$$

and according to eq.(4.9) act as follows:

$$Q_u |n_B, n_F\rangle \propto |n_B - 1, n_F + 1\rangle, \quad Q_u^+ |n_B, n_F\rangle \propto |n_B + 1, n_F - 1\rangle. \quad (4.14)$$

## 5 SUSY quantum mechanics on the given internal world

The quantum mechanical system of previous section is described by the self-adjoint Hamiltonian

$$H = \{Q_u, Q_u\} = Q_u^2 \begin{pmatrix} b b^+ & 0 \\ 0 & b^+ b \end{pmatrix} = \begin{pmatrix} H_u^+ & 0 \\ 0 & H_u^- \end{pmatrix}, \quad (5.1)$$

provided by

$$H_{u\pm} = \frac{1}{2} Q_u^2 \left[ -\frac{\partial^2}{\partial u^2} + W^2(u) \pm W'(u) \right] \geq 0, \quad (5.2)$$

which are a standard Schrödinger operators of the conventional SUSY quantum mechanics (see e.g. [64-79] and references therein) acting on  $L^2(\mathcal{R})$ . The eq.(4.13) and eq.(5.1) provide the conservation of the supercharge and the non-negativity of the Hamiltonian

$$\left[ H_u, Q_u \right] = \left[ H_u, Q_u^+ \right] = 0, \quad H_u \geq 0. \quad (5.3)$$

Replacing the operators eq.(5.3) with a new ones

$$Q_{u1} = Q_u + Q_u^+, \quad Q_{u2} = Q_u - Q_u^+, \quad (5.4)$$

one gets the superalgebra

$$\left\{ Q_{ui}, Q_{uk} \right\} = 2\delta_{ik} H_u, \quad \left[ Q_u, H_u \right] = 0. \quad (5.5)$$

A multiplicity of degeneracy of the levels of Hamiltonian  $H_u$  with the energy  $E_u$  equals to a dimension of invariant subspace with respect to the action of all the  $Q_u$ . If  $E_u = 0$ , then

the corresponding subspace is one-dimensional - a level of zero point energy. In general, the superalgebra eq.(5.5) with an arbitrary number of the supercharge operators  $Q_{u_i}$  ( $i = 1, \dots, N$ ) defines the Clifford algebra with the basis of  $q_{u_i} = Q_{u_i} / \sqrt{E_u}$  for nonzero energy levels of  $H_u$ , which remains a central point in the SUSY theories. Due to it a definition of the multiplicity of degeneracy of the levels reduced to a definition of a dimension of the representations of the Clifford algebra, which is well known. For the even and odd number  $N$  a dimension of the representation of Clifford algebra is given by the formula

$$\nu = 2^n = 2^{[N/2]}, \quad (5.6)$$

where  $[\dots]$  means the integer part, namely the  $\nu$  defines a number of states in given supermultiplet. Thus, incompatibility relations eq.(2.6.2) yields the major law for the supermultiplets that each of them contains an equal number of fermion and boson degrees of freedom

$$n_B = n_F. \quad (5.7)$$

Certainly, considering the operator  $(-1)^{2S}$ , where  $S$  is the spin angular momentum having eigenvalue  $+1$  acting on a bosonic state and eigenvalue  $-1$  acting on a fermionic state, by means of eq.(5.1) one gets

$$\begin{aligned} \sum_i \langle i | (-1)^{2S} H_u | i \rangle &= \sum_i \langle i | (-1)^{2S} Q_u Q_u^+ | i \rangle + \sum_i \langle i | (-1)^{2S} Q_u^+ Q_u | i \rangle = \\ \sum_i \langle i | (-1)^{2S} Q_u Q_u^+ | i \rangle - \sum_j \langle j | (-1)^{2S} Q_u Q_u^+ | j \rangle &= 0. \end{aligned} \quad (5.8)$$

Here one has used the relation of completeness  $\sum_i | i \rangle \langle i | = 1$  within the subspace of states invariant with respect to the action of  $Q_u$  and  $Q_u^+$ , and the fact that the operator  $(-1)^{2S}$  must anticommute with  $Q_u$ . The  $\sum_i \langle i | (-1)^{2S} H_u | i \rangle = E_u \text{Tr} [(-1)^{2S}]$  is proportional to the number of bosonic degrees of freedom  $n_B$  minus the number of fermionic degrees of freedom  $n_F$  in the trace. Hence, the eq.(5.7) holds for any  $E_u \neq 0$  in each supermultiplet.

The concept of shape invariance [66] in SUSY quantum mechanics has proven to be very useful because it leads immediately to exactly solvable potentials, namely a subset of the potentials for which the Schrödinger equations are exactly solvable share an integrability condition, while the partner potentials

$$V_1(u) = W^2(u) - \frac{Q_0}{\sqrt{2}} W'(u), \quad V_2(u) = W^2(u) + \frac{Q_0}{\sqrt{2}} W'(u), \quad (5.9)$$

satisfy the condition of shape invariance

$$V_2(u; a_1) = V_1(u; a_2) + R(a_1), \quad (5.10)$$

where  $a_{1,2}$  are a set of parameters specifying space-independent properties of the potentials. Therefore, the Hamiltonian eq.(5.2) can be readily factorized  $H_u - E_{u_0} = \hat{A}^+ \hat{A}$ , where

$E_{u_0}$  is the ground state energy, and

$$\hat{A} = W(u) + \frac{i Q_0}{\hbar \sqrt{2}} \hat{P}_u, \quad \hat{A}^+ = W(u) - \frac{i Q_0}{\hbar \sqrt{2}} \hat{P}_u. \quad (5.11)$$

The ground state wavefunction of  $H_u$  either belongs to  $H_{u+}$  or  $H_{u-}$

$$\psi_0^\pm(u) = \psi_0^\pm(\theta) \exp\left(\pm \int_0^u W(u) du\right) \quad (5.12)$$

satisfies the condition  $\hat{A} | \psi_0 \rangle = 0$ . The shape invariance has an underlying algebraic structure [75]. Depending on the asymptotic behavior of the superpotential  $W(u)$  one of the two functions  $\psi_0^\pm$  will be normalizable (good SUSY) or both are not be normalizable (broken SUSY). Clearing up this situation the Witten index [65] is turned out to be one of the useful tool which, according to the Atiyah-Singer index theorem [79,80], associates with the operator index. The latter is a topological characteristic and does not vary with the variation of the parameters of theory. Thus, the Witten index

$$\Delta(\beta) = \text{Tr} \left( P e^{-\beta H_u} \right), \quad \beta > 0, \quad (5.13)$$

depends only on the asymptotic values of  $W(u)$ . Here a self-adjoint operator  $P = P^+$ , called Witten parity, which anticommutes with the supercharges, and therefore commutes with the Hamiltonian, i.e.

$$\{Q_i, P\} = 0, \quad [H, P] = 0, \quad P^2 = 1,$$

explicitly can be written  $P = I \otimes \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ . Then, if  $\Delta(\beta) \neq 0$  one has a good SUSY.

## 6 $n \geq 1$ MW-SUSY on the internal worlds

Hereinafter we shall use the four-dimensional Weyl spinor notation for the  $n$  supersymmetry generators  $Q_\alpha^A (A = 1, \dots, n; \alpha = 1, 2)$ . The relativistic generalization of eq.(5.1)-eq.(5.5) for the  $n \geq 1$  MW-SUSY algebra with the complex scalar central charges  $X^{AB}$  defined on the given  $i$ -th ( $i=1, \dots, N$ ) internal manifold  $G_{u_i}$  reads

$$\begin{aligned} \{Q_{u_i}^A, \bar{Q}_{u_i \dot{\beta} B}\} &= 2 \, {}^i\sigma_{u_i \alpha \dot{\beta}}^{(\lambda, \delta)} P_{u_i}^{(\lambda, \delta)} \delta_B^A, \quad \{Q_{u_i}^A, Q_{u_i}^B\} = \epsilon_{\alpha\beta} X^{AB}, \\ \left[ Q_{u_i}^A, P_{u_i}^{(\lambda, \delta)} \right] &= \left[ \bar{Q}_{u_i \dot{\alpha} A}, P_{u_i}^{(\lambda, \delta)} \right] = \left[ P_{u_i}^{(\lambda, \delta)}, P_{u_i}^{(\tau, \rho)} \right] = 0. \end{aligned} \quad (6.1)$$

Here we have used the MW-notation, namely,  $\lambda = \pm, \delta = 1, 2, 3$ ,  $\epsilon_{\alpha\beta}$  is the 2-dimensional Levi-Civita symbol, and imposed the convention  $Q_{u_1}^A = \dots = Q_{u_N}^A \equiv Q_u^A$ . The matrices

${}^i\sigma_u^{(\lambda, \delta)}$  read

$${}^i\sigma_u^{(\pm\delta)} = \frac{1}{\sqrt{2}} \left( \kappa^\delta \sigma_i^0 \pm \sigma_i^\delta \right), \quad (6.2)$$

where  $\langle \kappa^\delta, \kappa^\rho \rangle = \delta^{\delta\rho}$ ,  $\sigma_i^m \equiv (\sigma_i^0, \vec{\sigma}_i)$  are the  $i$ -th sample of the Pauli matrices such that  $\bar{\sigma}_i^m \equiv (\sigma_i^0, -\vec{\sigma}_i)$ , and

$$\{\sigma_i^m, \sigma_j^n\} = \{\sigma_i^m, \bar{\sigma}_j^n\} = \{\bar{\sigma}_i^m, \bar{\sigma}_j^n\} = 0, \quad \text{if } i \neq j.$$

To trace a maximal resemblance in outward appearance to the conventional SUSY theories [24-59] within the next two sections we omit the subscript ( $u_i$ ) and the MW-index (i) to the left in eq.(6.1), denote the 6-dimensional vector index by the  $m \equiv (\lambda, \delta)$ . But it goes without saying that all the results obtained refer to the given 6-dimensional internal world  $G_{u_i}$ . The resulting mathematical structure of eq.(6.1) is closely similar to those of conventional SUSY algebra, although not identical. Thus, adopting a standard technique in the following it is worth briefly recording some questions but we shall forbear to write them out in details as they are so standard. We start with a discussion of the representations of SUSY on asymptotic single particle states. The Casimirs for  $n = 1$  MW-SUSY (the extension to  $n > 1$  is straightforward) are  $P^2 = P_m P^m$  and  $C^2 = C_{mn} C^{mn}$ , where  $C_{mn} = B_M P_n - B_n P_m$ ,  $B_m = W_m - \frac{1}{4} \bar{Q}_{\dot{\alpha}} \bar{\sigma}_m^{\dot{\alpha}\beta} Q_\beta$ ,  $W_m$  is the Pauli-Ljubanski vector, which has eigenvalues  $-m^2 s(s+1)$ ,  $s = 0, \frac{1}{2}, 1, \dots$  for massive states, and  $W_m = \lambda P_m$  for massless states with the helicity  $\lambda$ . For the  $n = 1$  MW-SUSY, no central charges, massive states (from the rest frame) the Clifford vacuum state  $|\Omega\rangle$  is actually an eigenstate of spin angular momentum  $|\Omega\rangle = m, s, s_3$ . Hence all the states in the MW-SUSY irrep are characterized only by the mass and spin. The normalized creation and annihilation operators are defined as follows:

$$a_{1,2} = \frac{1}{2m} Q_{1,2}, \quad a_{1,2}^+ = \frac{1}{2m} \bar{Q}_{1,2}, \quad (6.3)$$

while for a given  $|\Omega\rangle$  the full massive MW-SUSY irrep contains the states  $|\Omega\rangle$ ,  $a_1^+ |\Omega\rangle$ ,  $a_2^+ |\Omega\rangle$  and  $\frac{1}{\sqrt{2}} a_1^+ a_2^+ |\Omega\rangle = -\frac{1}{\sqrt{2}} a_2^+ a_1^+ |\Omega\rangle$ . There are a total of  $4(2j+1)$  states in the massive irrep, where  $j$  equal integers or half-integers. In the fundamental  $n = 1$  MW-SUSY massive irrep ( $j = 0$ ) there are four states with  $S_3 = 0, -\frac{1}{2}, \frac{1}{2}$  and  $0$ , respectively. Taking into account that the parity operation interchanges  $a_1^+$  and  $a_2^+$ , then the fundamental massive irrep contains one massive Weyl fermion, one real scalar and one real pseudoscalar. In the same manner one gets for the massless states of the  $n = 1$  MW-SUSY, no central charges, that the  $|\Omega\rangle$  is nondegenerate and has definite helicity  $\lambda$ - there is only one pair of creation and annihilation operators ( $\bar{Q}_2 \equiv 0$ ) and the massless  $n = 1$  MW-SUSY irrep contains two states  $|\Omega\rangle$  with helicity  $\lambda$  and  $\lambda + \frac{1}{2}$ . Such an analysis can be readily extended to the massive and massless states of the  $n > 1$  MW-SUSY, no central charges [26,35,41]. For example, there are  $2^{2n}(2j+1)$  states in a massive irrep, while the helicity in the massless irrep are  $\lambda, \lambda = \frac{1}{2}, \dots, \lambda + \frac{n}{2}$ .

In the presence of the central charges with the Wess and Bagger convention ( $X^{AB} = -X_{AB}$ ) the basis to describe them must be re-diagonalized using the Zumino's decomposition for the massive state in the rest frame, where the eigenvalues of the central charges  $Z_M$  are real and non-negative. As far as the  $2n$  pairs of creation and annihilation operators  $\{a, a^+\}$  and  $\{b, b^+\}$  are positive definite operators and  $Z_M$  are non-negative,

then: 1) in any MW-SUSY irrep  $Z_M \leq 2m$ ; 2) if  $Z_M \leq 2m$  then the multiplicities of the massive irreps are the same as the case of no central charges; 3) if one saturates the bound, i.e.  $Z_M = 2m$  for some or all  $Z_M$ , there are the reduced massive multiplets called short multiplets and the states are referred to as BPS- saturated states after the BPS monopoles in SUSY gauge theories. The MW-SUSY irreps, like the conventional SUSY, on asymptotic single particle state will automatically carry a representation of the automorphism group  $(Q_{\alpha}^A \rightarrow U^A_B Q_{\alpha}^B, \bar{Q}_{\dot{\alpha}A} \rightarrow \bar{Q}_{\dot{\alpha}B} U^{+B}_A)$ . For massless irreps  $U(N)$  is the largest automorphism symmetry which respects helicity, while the same for massive irreps which respects spin is the unitary symplectic group of rank  $N$  -  $USp(2N)$ . In the presence of central charges, if none of them saturates the BPS bound, the automorphism group still remains  $USp(2N)$ . At last, if one central charge saturates the BPS bound, the automorphism group reduces to  $USp(N)$  for  $N$  even, or  $USp(N+1)$  for  $N$  odd. The automorphism symmetries impose some constraints on the internal symmetry group. When the MW-SUSY is represented on quantum fields, instead of asymptotic states, the Clifford vacuum condition reads

$$[\bar{Q}_{\dot{\alpha}}, |\Omega\rangle] = 0, \quad (6.4)$$

where  $|\Omega\rangle$  must be a complete scalar field.

## 7 Coset construction and $n = 1$ rigid internal manifold $SG_u$

A universal geometric description of field systems respecting the invariance under a symmetry group  $G$  is provided by the method of nonlinear or the coset space realizations. One considers  $G$  as a group of transformations acting in some coset space with an properly chosen stability group  $H$  and identifies the coset parameters with fields. In conventional SUSY theories is widely used the superspace- superfield formalism, i.e. the manifestly supersymmetric technique for constructing superfields carried out by a generalization of the coset construction [81-93]. Introducing constant Grassmann spinors  $\theta^\alpha, \bar{\theta}_{\dot{\alpha}}$ , one rewrites the  $n = 1$  MW-SUSY algebra eq.(6.1) as a Lie algebra of supergroup  $G$ , where the Maurer-Cartan form is valued. One constructs the coset  $G/H$   $\Omega \sim \Omega h$ ,  $\Omega \in G$ ,  $h \in H$ , where  $H$  is the stability group closing the generators of unbroken 6-dimensional rotations in  $G_u$  and unbroken internal symmetries. Given a Lie algebra one can exponentiate to yield the general group element

$$\Omega(u, \theta, \bar{\theta}, \omega) = e^{i(-u^m P_m + \theta Q + \bar{\theta} \bar{Q})} e^{-\frac{1}{2} \omega^{mn} M_{mn}}, \quad (7.1)$$

where one chooses to keep all of  $G$  unbroken ( $h = 1$ ). Whence it is clear that  $Z_u^A \equiv (u, \theta, \bar{\theta})$  parametrizes a  $(6 + 4)$ -dimensional coset space:  $n = 1$  rigid (global SUSY) internal supermanifold  $SG_u$  (internal super world). A SUSY transformation is specified by the group element

$$g = e^{i(\xi^\alpha Q_\alpha + \bar{\xi}_{\dot{\alpha}} \bar{Q}^{\dot{\alpha}})} \quad (7.2)$$

provided by conventional Grassmann parameters  $(\xi, \bar{\xi})$ . The differential operators  $Q$  and  $\bar{Q}$  closed into the MW-SUSY algebra and explicitly read in standard manner. The Maurer-

Cartan form  $\Omega^{-1} \Omega$  transforms under a rigid group  $G$

$$\Omega \rightarrow g \Omega h^{-1}, \quad \Omega^{-1} d\Omega \rightarrow h (\Omega^{-1} d\Omega) h^{-1} - dh h^{-1}, \quad (7.3)$$

which induces the motion ( $n = 1$ )

$$\begin{aligned} u^m &\rightarrow u^m + i\theta\sigma^m\bar{\xi} - i\xi\sigma^m\bar{\theta}, \\ \theta^\alpha &\rightarrow \theta^\alpha + \xi^\alpha, \quad \bar{\theta}_{\dot{\alpha}} \rightarrow \bar{\theta}_{\dot{\alpha}} + \bar{\xi}_{\dot{\alpha}}. \end{aligned} \quad (7.4)$$

Computing the Maurer-Cartan form one can extract the covariant vielbein and the  $H$  connection  $\omega^i = 0$ . The  $(6 + 4)$ -dimensional vielbein  $E_M^A$  and spin connection  $W_A^{mn}$  enable to write the general form of covariant derivative in a such a space

$$D_M = E_M^A (\partial_A + \frac{1}{2} W_A^{mn} M_{mn}), \quad (7.5)$$

where  $M$  is to denote an superspace index,  $A$  is the super-tangent space index. With these building blocks it is readily to treat in fairly conventional manner a whole formalism of  $n = 1$  superfields and total supersymmetric actions invariant under the group  $G$ .

## 8 Broken super operator multimanifold $\hat{\mathcal{S}}G_N$

Following [2] (see App.A) the frame field of the particles (left-handed leptons and quarks) defined on the MW geometry  $G_N = G \oplus_{\eta} G \oplus_{u_1} \cdots \oplus_{u_N} G$  can be written

$$\chi_p = \chi_p(\eta) \chi_{u_p}(u), \quad \chi_{u_p}(u) \equiv \chi_{u_1}(u_1) \cdots \chi_{u_N}(u_N). \quad (8.1)$$

Here  $\chi_{u_i}$  is the left-handed Weyl spinor field defined on the manifold where the subquarks of corresponding species are involved, the suffix  $p$  refers to particle. To develop some feeling for our approach and to avoid irrelevant complications here temporarily we skeletonize the microscopic structure of the particles presented it in complete generality, but in the subsec.12.4 and 12.5 we shall specialize to the case of leptons and quarks. To reproduce a supersymmetric generalization of this model the idea is now to promote  $G_{u_i}$  into the  $n = 1$  rigid internal supermanifold  $SG_{u_i}$ . Accordingly, each left-handed fermionic  $\chi_{u_i}(u_i)$  component must be promoted to chiral multiplets by adding superpartner as required by SUSY. Corresponding to  $\chi_{u_i}(u_i)$  the chiral superfield  $\tilde{\Phi}_{ch}(u_i)$  contains off-shell four real fermionic degrees of freedom ( $n_F = 4$ ) and four real bosonic degrees of freedom ( $n_B = 4$ ), namely, it contains a left-handed Weyl fermion  $\chi_{p\alpha}(u_i)$  of spin  $\frac{1}{2}$  and mass dimension  $\frac{3}{2}$ , a complex scalar field  $A_{sp}(u_i)$  of spin 0 and mass dimension 1, and an auxiliary complex scalar field  $F(u_i)$  of spin 0 and mass dimension 2 (a suffix (s) denotes a sparticle):  $\tilde{\Phi}_{u_i}(u_i) = (\chi_{p\alpha}(u_i), A_{sp}(u_i), F(u_i))$ . Since the chain rule holds, i.e., any product of chiral superfields is also a chiral superfield, then the  $u$ -component in eq.(8.1) can be rewritten as follows:

$$\chi_u(u) \rightarrow \tilde{\Phi}_{ch}(u) = \tilde{\Phi}_{ch}(u_1) \cdots \tilde{\Phi}_{ch}(u_N), \quad (8.2)$$

with the component fields

$$\tilde{\Phi}_u^{ch}(u) = \left( \chi_{p\alpha}(u), A_{sp}(u), F(u) \right), \quad (8.3)$$

provided by the following MW representations

$$\begin{aligned} \chi_{p\alpha}(u) &= \left[ \chi_{u_1}(u_1) \cdots \chi_{u_N}(u_N) \right]_{\alpha}, \quad A_{sp}(u) = A_{u_1 sp}(u_1) \cdots A_{u_N sp}(u_N), \\ F(u) &= F(u_1) \cdots F(u_N). \end{aligned} \quad (8.4)$$

Accordingly, the particle field  $\chi_p(\zeta)$  in eq.(8.1) should be promoted to the  $\tilde{\Phi}_{\not{ch}}$  field defined on the broken super MM ( $\$MM$ )

$$\not{S}G_N = G \oplus \not{S}G_N \equiv G \oplus \not{S}G_{u_1} \oplus \cdots \oplus \not{S}G_{u_N}, \quad (8.5)$$

where the suffix ( $\$$ ) denotes a broken SUSY. The  $\eta$ -component of the particle breaks the whole symmetry between the frame superfields of particle and sparticle, since the  $\eta$ -component of the sparticle is absent:

$$\chi_p(\zeta) \rightarrow \tilde{\Phi}_{\not{ch}} = \left( \chi_p(\eta) \chi_p(u), A_{sp}(u), F(u) \right). \quad (8.6)$$

Certainly, note that due to subquark algebra eq.(4.5) the MW-SUSY has arisen only on the internal worlds where the subquarks are defined. By this we arrive to the major point for an understanding the principle fact why there is no any direct experimental evidence for the existence of any of the numerous sparticles predicted by SUSY. Our view point is as follows:

- *the sparticles merely could not survive on the manifold  $G_{\eta}$ , which clearly means that they always would be absent in the Minkowski spacetime continuum (sec.2.1).*

To see the nature of the supersymmetric generalization involved, it prompt us further to introduce a new mathematical framework of the  $\$OMM$ :  $\hat{\not{S}}G_N$ . The first step towards it is to promote the given OM:  $\hat{G}_{u_i}$  into the internal super operator manifold (SOM)  $\hat{\not{S}}G_{u_i}$ . According to general scheme given in sec.2, it implies that the operator vectors  $\hat{\Psi}_u = {}^i\hat{\gamma}_m {}^i\psi_u^m$ , ( $m = (\lambda, \delta), \lambda = \pm, \delta = 1, 2, 3$ ) of a tangent section of principle bundle with the base  $\hat{G} = \hat{G}_{u_1} \oplus \cdots \oplus \hat{G}_{u_N}$  are promoted to the MW super operator vectors  $\hat{\Phi}_u(Z_u)$  given by the expansion

$$\hat{\Psi}_u \rightarrow \hat{\Phi}_u(Z_u) = {}^i\hat{\gamma}_M {}^i\tilde{\Phi}_u^M(Z_u) \left( \in \hat{\not{S}}G = \hat{\not{S}}G_{u_1} \oplus \cdots \oplus \hat{\not{S}}G_{u_N} \right), \quad (8.7)$$

which has arisen by the corresponding substitution

$${}^i\hat{\gamma}_m \rightarrow {}^i\hat{\gamma}_M = \left( {}^i\hat{\gamma}_m, a_{\alpha}, a_{\alpha}^+ \right), \quad {}^i\psi_u^m \rightarrow {}^i\tilde{\Phi}_u^M(Z_u) = \left( {}^i\psi_u^m, \psi^{\mu}, \bar{\psi}^{\dot{\mu}} \right), \quad (8.8)$$

where  $M$  is the superspace index, the annihilation and creation operators  $a, a^+$  are given in eq.(6.3), also we take into account that in MW-SUSY algebra eq.(6.1) the convention  ${}^1Q_u = \cdots = {}^N Q_u \equiv Q$  is imposed, and introduce required spinor components

$$\psi^{\mu} = \theta^{\mu} \psi(\theta), \quad \bar{\psi}^{\dot{\mu}} = \bar{\theta}^{\dot{\mu}} \bar{\psi}(\bar{\theta}). \quad (8.9)$$



Hence

$$\hat{\hat{\Phi}}_u(Z_u) = \hat{\Psi}_u + \hat{\psi} + \hat{\bar{\psi}}, \quad (8.10)$$

where

$$\hat{\psi} = \hat{\theta}\psi(\theta), \quad \hat{\bar{\psi}} = \hat{\bar{\theta}}\bar{\psi}(\bar{\theta}), \quad \hat{\theta} = a\theta, \quad \hat{\bar{\theta}} = a^+\bar{\theta}. \quad (8.11)$$

Given the SOM:  $\hat{S}_u^G$  we can extend it to  $\mathcal{S}$ OMM:  $\hat{\mathcal{S}}G_N$ :

$$\hat{\mathcal{S}}G_N = \hat{G}_\eta \oplus \hat{S}_u^G = \hat{G}_\eta \oplus \hat{S}_{u_1}^G \oplus \cdots \oplus \hat{S}_{u_N}^G. \quad (8.12)$$

The super operator vectors read

$$\begin{aligned} \hat{\hat{\Phi}}_{\mathcal{S}}(Z) &= \begin{cases} \hat{\Psi}_\eta p(\eta) + \hat{\hat{\Phi}}_u(Z_u) & \text{for particle,} \\ \hat{\hat{\Phi}}_u(Z_u) & \text{for sparticle,} \end{cases} = \\ \hat{\Phi} + \hat{\psi} + \hat{\bar{\psi}} &= \begin{cases} \hat{\Phi}_p(\zeta) + \hat{\psi} + \hat{\bar{\psi}} & \text{for particle,} \\ \hat{A}_{sp}(u) + \hat{\psi} + \hat{\bar{\psi}} & \text{for sparticle,} \end{cases} \end{aligned} \quad (8.13)$$

where

$$\begin{aligned} \hat{\Phi}_p(\zeta) &= \hat{\Psi}_\eta p(\eta) + \hat{\Psi}_u p(u) \in \hat{G}_N, \quad \hat{A}_{sp}(u) \equiv \hat{A}_u{}_{sp}(u) \in \hat{G}_N, \\ \hat{\Psi}_\eta p(\eta) &= \hat{\gamma}_m \psi^m(\eta). \end{aligned} \quad (8.14)$$

It is remarkable that such a MW-SUSY generalization does not upset a major condition of the MW geometry  $G_N$  realization on which the MW frame fields of particles are defined [2] (subsec.12.6). Certainly, taking into account that  $\theta$  and  $\bar{\theta}$  are Grassmann variables, a straightforward computation gives the important MW-SUSY generalization of matrix element of the bilinear form eq.(3.1.1)

$$\begin{aligned} \sum_{\lambda=\pm} < \tilde{\chi}_\lambda | \hat{\hat{\Phi}}_{\mathcal{S}}(Z) \hat{\hat{\Phi}}_{\mathcal{S}}(Z) | \tilde{\chi}_\lambda > &= \sum_{\lambda=\pm} < \tilde{\chi}_\lambda | \hat{\hat{\Phi}}_{\mathcal{S}}(Z) \hat{\hat{\Phi}}_{\mathcal{S}}(Z) | \tilde{\chi}_\lambda > = \\ \sum_{\lambda=\pm} < \chi_\lambda | \hat{\Phi}(\zeta) \hat{\bar{\Phi}}(\zeta) | \chi_\lambda > &= -i \lim_{\zeta \rightarrow \zeta'} (\zeta \zeta') G(\zeta - \zeta') = \\ \begin{cases} -i \left( \eta^2 G_\eta(0) - \sum_{i=1}^N u_i^2 G_{u_i}(0) \right) & \text{for particle,} \\ i \sum_{i=1}^N u_i^2 G_{u_i}(0), & \text{for sparticle,} \end{cases} \end{aligned} \quad (8.15)$$

provided by the corresponding Green's functions and generalized state vectors

$\tilde{\chi}_\lambda = (\chi_\lambda, | \Omega >)$ , where  $| \Omega >$  is the Clifford vacuum. The  $G_N$ -realization condition readily stems from the requirement that the bilinear form eq.(8.15) as a geometric object should be finite

$$\left| \sum_{\lambda=\pm} < \tilde{\chi}_\lambda | \hat{\hat{\Phi}}_{\mathcal{S}}(Z) \hat{\hat{\Phi}}_{\mathcal{S}}(Z) | \tilde{\chi}_\lambda > \right| = \left| \sum_{\lambda=\pm} < \chi_\lambda | \hat{\Phi}(\zeta) \hat{\bar{\Phi}}(\zeta) | \chi_\lambda > \right| < \infty. \quad (8.16)$$

As it can be seen from eq.(8.15), the latter may be satisfied only for the particles but not for sparticles. For particles it reduced to

$$\zeta^2 G_\zeta(0) = \eta^2 G_\eta(0) - \sum_{i=1}^N u_i^2 G_{u_i}(0) < \infty. \quad (8.17)$$

The particle should be on the mass shell if

$$m \equiv |p_u| \equiv \left( \sum_{i=1}^N p_{u_i}^2 \right)^{1/2} = |p_\eta|, \quad (8.18)$$

i.e. the MW geometry  $G_N$  comes into being (sec.3) if

$$G_\zeta^p(0) = G_\eta^p(0) = G_u^p(0), \quad u_p^2 G_p^u(0) \equiv \sum_{i=1}^N u_i^2 G_{u_i}^p(0). \quad (8.19)$$

## 9 Realistic realization of the $n \geq 1$ MW-SUSY

Our major goal is to develop a realistic MW-SUSY extension of the MSM [2], which is not convenient to carry out directly within considered scheme, where the broken MW-SUSY is defined on the  $\mathcal{S}MM$ :  $\mathcal{S}G_N$  eq.(8.5). However, much easier to handle it at first within the exact MW-SUSY defined on the whole MW coset manifold

$$SG_N = SG_\eta \oplus SG_u \oplus \dots \oplus SG_{u_N}, \quad (9.1)$$

and afterwards to get back to the broken MW-SUSY. The MW coset manifold eq.(9.1) can be restored by *lifting up* the manifold  $G_\eta$  to supermanifold  $SG_\eta$ . Then, we introduce at once the corresponding supercharge operators  $Q_\eta = Q_0 b \otimes f^+$  and  $Q_\eta^+ = Q_0 b^+ \otimes f$  (eq.(4.12)). According to eq.(4.5) one has  $Q_\eta = \frac{1}{\sqrt{2}} \frac{q_0^2}{\bar{q}_0} = i Q_{u_1} = \dots i Q_{u_N}$ . In the same time one must *lift up* also each sparticle to corresponding particle state by assigning to it the  $A_{sp}(\eta)$  component defined on  $G_\eta$  in order to provide the mass for sparticle equaled exactly to the mass of corresponding particle. It will enable the sparticle to be included in the same supermultiplet with corresponding particle. Then, the MW frame field of sparticle eq.(8.4) can be rewritten as follows:

$$A_{u,sp}(u) \rightarrow A_{sp}(\zeta) = A_{sp}(\eta) A_{u,sp}(u), \quad (9.2)$$

while according to eq.(8.18) one gets

$$p_{\eta,sp} = p_{\eta,p} = m = p_{u,p} = p_{u,sp}. \quad (9.3)$$

Given eq.(9.3) now the sparticles like particles are free to emerge in  $G_\eta$  because of a validity of the condition imposed upon the Green's function  $G_{\eta,sp}(0) = G_{u,sp}(0)$ . Hence, we can write down the first of the anticommutation relations given in eq.(6.1) for the exact  $n \geq 1$  MW SUSY algebra

$$\begin{aligned} \{Q_\alpha^A, \bar{Q}_{\dot{\beta}B}\} &= 2 \, {}^i \sigma_{\eta\alpha\dot{\beta}}^{(\lambda,\delta)} P_{(\lambda,\delta)} \delta_B^A, \\ \{Q_{u_1\alpha}^A, \bar{Q}_{\dot{\beta}B}\} &= 2 \, {}^i \sigma_{u_1\alpha\dot{\beta}}^{(\lambda,\delta)} P_{(\lambda,\delta)} \delta_B^A, \\ &\dots\dots\dots \\ \{Q_{u_N\alpha}^A, \bar{Q}_{\dot{\beta}B}\} &= 2 \, {}^i \sigma_{u_N\alpha\dot{\beta}}^{(\lambda,\delta)} P_{(\lambda,\delta)} \delta_B^A. \end{aligned} \quad (9.4)$$

Summing over all them we get the first anticommutation relation of the exact  $n \geq 1$  MW SUSY algebra defined on the SMM:  $SG_N$ :

$$\{Q_\alpha^A, \bar{Q}_{\dot{\beta}B}\} = 2 \, {}^i\sigma_{\alpha\dot{\beta}}^m \, {}^iP_m \delta_B^A. \quad (9.5)$$

Here  $Q \equiv \sqrt{N} Q_\eta = i \sqrt{N} Q_u$ ,  $Q_0 \equiv \sqrt{N} Q_0 = i \sqrt{N} Q_u$ , and having used the MW- notations, i.e., now  $m \equiv (\lambda, \mu, \delta)$  is the 12-dimensional vector index where we let the first two subscripts in the parentheses to denote the pseudovector components while the third refers to the ordinary vector components,  ${}^iP_m$  is the 12-dimensional momentum defined on the given internal world  $G$ . The matrices  ${}^i\sigma^m$  read

$$\begin{aligned} {}^i\sigma^{(1,1,\delta)} &= \frac{1}{\sqrt{2}} \left( \nu_i \sigma_\eta^{(+\delta)} + {}^i\sigma_u^{(+\delta)} \right), & {}^i\sigma^{(2,1,\delta)} &= \frac{1}{\sqrt{2}} \left( \nu_i \sigma_\eta^{(+\delta)} - {}^i\sigma_u^{(+\delta)} \right), \\ {}^i\sigma^{(1,2,\delta)} &= \frac{1}{\sqrt{2}} \left( \nu_i \sigma_\eta^{(-\delta)} + {}^i\sigma_u^{(-\delta)} \right), & {}^i\sigma^{(2,2,\delta)} &= \frac{1}{\sqrt{2}} \left( \nu_i \sigma_\eta^{(-\delta)} - {}^i\sigma_u^{(-\delta)} \right), \end{aligned} \quad (9.6)$$

where

$${}^i\sigma_\eta^{(\pm\delta)} = \frac{1}{\sqrt{2}} \left( \kappa^\delta \sigma_\eta^0 \pm \sigma_\eta^\delta \right), \quad (9.7)$$

for  $\nu_i$  see the eq.(3.2), the  $\kappa^\delta$  and matrices  $\sigma^m$  are given in eq.(6.2). In the same manner we get the rest of the MW-SUSY relations. The resulting exact  $n \geq 1$  MW-SUSY, central charges, algebra defined on the SMM:  $SG_N$  reads

$$\begin{aligned} \{Q_\alpha^A, \bar{Q}_{\dot{\beta}B}\} &= 2 \, {}^i\sigma_{\alpha\dot{\beta}}^m \, {}^iP_m \delta_B^A, \quad \{Q_\alpha^A, Q_\beta^B\} = \epsilon_{\alpha\beta} X^{AB}, \quad m \equiv (\lambda, \mu, \delta), \\ [Q_\alpha^A, {}^iP] &= [\bar{Q}_{\dot{\alpha}A}, {}^iP_m] = [{}^iP_m, {}^iP_n] = 0, \\ [Q_\alpha^A, {}^iM_{mn}] &= {}^i\sigma_{mn} \alpha^\beta Q_\beta^B, \quad [\bar{Q}_{\dot{\alpha}A}, {}^iM_{mn}] = {}^i\sigma_{mn}^{\dot{\alpha}\dot{\beta}} \bar{Q}_{\dot{\beta}A}, \end{aligned} \quad (9.8)$$

provided by  $X^{AB} \equiv \sqrt{N} X_\eta^{AB} = i \sqrt{N} X_u^{AB}$ . Due to different features of particles and sparticles in the physically realistic framework of eq.(8.15), one must have always to distinguish them, especially, for obtaining the realistic theory by passing back to the physical limit  $A_{sp}(\zeta) \rightarrow A_{sp}(u)$  (see eq.(9.11)). Only natural way to do that is to formulate conservation law by introducing an additional discrete internal symmetry, which assigns the conserved charges  $+1$  to the particles and  $-1$  to the sparticles. This is a major motivation of a new symmetry due to which the sparticles can only be produced in pairs. The Jacobi identity for  $[[Q, B], B]$  ( $B$  are the generators of internal symmetry group) implies that the structure constants vanish, i.e., the internal symmetry algebra is Abelian. Only one independent combination of the Abelian generators actually has a nonzero commutator with  $Q$  and  $\bar{Q}$  ( $n = 1$ ). This  $U(1)$  generator denoted by  $R$

$$[Q_\alpha, R] = Q_\alpha, \quad [\bar{Q}_{\dot{\alpha}}, R] = -\bar{Q}_{\dot{\alpha}} \quad (9.9)$$

is known in conventional SUSY theories as the  $R$ -parity. Thus, the  $n = 1$  MW SUSY algebra in general possesses an internal (global)  $U(1)$  symmetry, for which SUSY generators have  $R$ -charge  $+1$  and  $-1$ , respectively. This is a multiplicative  $Z_2$  discrete symmetry defined as

$$R = (-1)^{3B+L+2S}, \quad (9.10)$$

where  $S$ ,  $B$  and  $L$  are the spin, the baryon and the lepton numbers (sec.12).  $R$ -parity leaves invariant the fields of the SM, and flips the sign of their superpartners. Therefore, it implies that sparticles are pair produced and the lightest sparticle cannot decay. Reflecting upon said above as the first step towards constructing the realistic theory we shall start with unbroken MW-SUSY implemented on the SMM:  $SG_N$  by lifting up all the sparticles to corresponding particle states provided by the  $R$ -parity conservation. Using then the conventional rules governing the MW-SUSY theory we may write the most generic renormalizable MW-SUSY action  $S_{SG_N}$  (sec.14) involving gauge and matter frame fields, and, thus, the corresponding generating functional  $Z_{SG_N}[\mathcal{J}]$ . Of course, it cannot be the *exact action* describing the *exact symmetry* in nature. Along this line it has to be realized in its broken phase, and at the second step the corresponding generating functional  $Z_{SG_N}[\mathcal{J}]$  should be defined on  $\mathcal{S}G_N$  by passing back to the physical limit  $A_{sp}(\zeta) \rightarrow \underset{u}{A}_{sp}(u)$  for all the sparticles involved which respects the  $R$ -parity:

$$Z_{real}[\mathcal{J}] \equiv Z_{\mathcal{S}G_N}[\mathcal{J}] = \lim_{\left(R; A_{sp}(\zeta) \rightarrow \underset{u}{A}_{sp}(u)\right)} Z_{SG_N}[\mathcal{J}]. \quad (9.11)$$

Such a breaking of the MW-SUSY can be implemented on  $\mathcal{S}G_N$  first of all by subtracting back all the explicit *soft* mass terms  $S_{soft}$  formerly introduced for the sparticles eq.(9.3)

$$S_{\mathcal{S}G_N} = S_{SG_N} + S_{soft}, \quad (9.12)$$

i.e., the mass terms of the scalar members of the chiral multiplets, apart from the Higgs chiral multiplet (sec.13), for which one must subtract the mass term of the Higgsino instead of scalar, and for the gaugino members of vector supermultiplets in the Lagrangian (sec.14). These soft terms do not reintroduce the quadratic diagrams [94], which motivated the introduction of SUSY. By this one has indeed returned to the  $\mathcal{S}G_N$

$$SG_{\eta} \left( R; A_{sp}(\zeta) \rightarrow \underset{u}{A}_{sp}(u) \right) \rightarrow G_{\eta}. \quad (9.13)$$

Hence, only the particles should be survived on  $G_{\eta}$ , but not the sparticles at all. The resulting formula eq.(9.11) can be regarded as the major recipe for a *realistic realization of any MW-SUSY* including the MW-supergravity and MW-superstrings. A remarkable feature of such an approach resides in the important fact that even if the MW-SUSY is broken, due to the splitting between the fermion-boson supersymmetric partners is itself of  $O(\text{VEV})$ , the naturalness problem in the MSM is resolved. Actually, the systems with the unbroken MW-SUSY defined on the  $SG_N$  are very constrained in the sense that the observables vary holomorphically with the parameters of theory. According to Cauchy's theorem, such complex analytic functions are determined in terms of very little data: the singularities and the asymptotic behavior on the hypersurface  $SG_N$ . It is then as well true a vice versa that the boson-fermion cancellation in the hierarchy problem can be thought to be a consequence of a constraint stemming from holomorphy and it will be held on the  $\mathcal{S}G_N$  as well at the limit eq.(9.11).

## 10 MW super operator multimanifold $\hat{SG}_N$

The structure of the resulting algebra eq.(9.8) is similar to those of eq.(6.1). The sole difference is that in eq.(9.8) instead of 6-dimensional vector indices now we use the 12-dimensional vector indices  $m = (\lambda, \tau, \delta)$ , where the first two in parenthesis denote the pseudovector components  $(\lambda, \tau = 1, 2)$  while the third refers to the ordinary vector components  $\delta = 1, 2, 3$ , and the double occurrence of dummy MW-indices to the left will be taken to denote a summation extended over all their values  $i = 1, \dots, N$ . Hence the Casimirs, the representations and the coset constructions of the exact MW-SUSY can be treated in similar manner of sec.6. For example, the Maurer-Cartan form is valued in a Lie algebra of supergroup  $G$  has a standard expansion

$$\Omega^{-1} d\Omega = i \left( \omega^A \Gamma_A + \omega^a \Gamma_a + \omega^i \Gamma_i \right), \quad (10.1)$$

where constructing the coset  $G/H$ , the generators  $\Gamma_i$  of unbroken 12-dimensional rotations in  $G_N$  and unbroken internal symmetries close into the stability group  $H$ ,  $\Gamma_A$  are the generators of unbroken translations in  $G_N$ ,  $\Gamma_i$  are the generators of spontaneously broken internal symmetries and symmetries of  $G_N$ . The  $\omega^A$ ,  $\omega^a$  and  $\omega^i$  are the set of one-forms on the SMM  $SG_N$  parametrized by the coordinates  ${}^i Z^A = ({}^i \zeta, \theta, \bar{\theta})$ . The general group element can be written

$$\Omega({}^i \zeta, \theta, \bar{\theta}, \omega) = e^{i({}^i \zeta^m {}^i P_m + \theta Q + \bar{\theta} \bar{Q})} e^{-\frac{1}{2} {}^i \omega^{mn} {}^i M_{mn}}. \quad (10.2)$$

As usual, one chooses to keep all of  $G$  unbroken. The  ${}^i Z^A$  parametrizes a  $N(6+4)$ -dimensional coset space  $SG_N$ . The transformation

$$\Omega \rightarrow g \Omega h^{-1} \quad (10.3)$$

( $h \in H$ ) induces the motion

$${}^i \zeta^m \rightarrow {}^i \zeta^m + i \theta {}^i \sigma^m \xi - i {}^i \xi \sigma^m \bar{\theta}, \quad \theta^\alpha \rightarrow \theta^\alpha + \xi^\alpha, \quad \bar{\theta}_{\dot{\alpha}} \rightarrow \bar{\theta}_{\dot{\alpha}} + \bar{\xi}_{\dot{\alpha}}, \quad (10.4)$$

and  $h = 1$ . The differential operators  $Q$  and  $\bar{Q}$  close into the  $n = 1$ , no central charges, MW-SUSY algebra

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2 {}^i \sigma_{\alpha\dot{\alpha}}^m {}^i \partial_m, \quad \{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0, \quad (10.5)$$

where

$$Q_\alpha = \frac{\partial}{\partial \theta^\alpha} - i {}^i \sigma_{\alpha\dot{\alpha}}^m \bar{\theta}^{\dot{\alpha}} {}^i \partial_m, \quad \bar{Q}^{\dot{\alpha}} = \frac{\partial}{\partial \bar{\theta}_{\dot{\alpha}}} - i \theta^\alpha {}^i \sigma_{\alpha\dot{\beta}}^m \epsilon^{\dot{\alpha}\dot{\beta}} {}^i \partial_m. \quad (10.6)$$

The covariant vielbein and covariant derivative respectively are

$$E_M^A = \begin{pmatrix} \delta_m^n & 0 & 0 \\ -i {}^i \sigma_{\mu\dot{\mu}}^n \bar{\theta}^{\dot{\mu}} & \delta_\mu^\alpha & 0 \\ -i \theta^\mu {}^i \sigma_{\mu\nu}^n \epsilon^{\nu\dot{\mu}} & 0 & \delta_{\dot{\alpha}}^{\dot{\mu}} \end{pmatrix} \quad (10.7)$$

and

$${}^i D_m = {}^i \partial_m, \quad D_\alpha = \frac{\partial}{\partial \theta^\alpha} + i {}^i \sigma_{\alpha\dot{\alpha}}^m \bar{\theta}^{\dot{\alpha}} {}^i \partial_m, \quad \bar{D}^{\dot{\alpha}} = \frac{\partial}{\partial \bar{\theta}_{\dot{\alpha}}} + i \theta^\alpha {}^i \sigma_{\alpha\dot{\beta}}^m \epsilon^{\dot{\alpha}\dot{\beta}} {}^i \partial_m. \quad (10.8)$$

The rest of the whole formalism of  $n = 1$  MW-SUSY irreps and their supersymmetric actions invariant under the group  $G$  can be treated by standard technique. The irreps contain an equal number of bosonic and fermionic degrees of freedom with degenerate mass, which is a generic feature of supersymmetric field theories stemming from the fact that  $P^2$  is a Casimir operator of SUSY algebra. Since  $Q$  has mass dimension  $1/2$  and thus the mass dimensions of the fields in the irrep differ by  $3/2$ . According to generic scheme given in sec.8, the  $\chi_p(i\zeta)$  field in eq.(8.1) should be promoted to the  ${}^i\tilde{\Phi}({}^iZ^A)$  defined on the SMM:  $SG_N: {}^i\tilde{\Phi}^M({}^iZ^A) = ({}^i\Phi^m({}^i\zeta), \psi^\mu(\theta), \bar{\psi}^{\dot{\mu}}(\bar{\theta}))$ , where  $\psi^\mu(\theta)$  and  $\bar{\psi}^{\dot{\mu}}(\bar{\theta})$  are given in eq.(8.9). The operator vectors  $\hat{\Phi}({}^i\zeta) = {}^i\hat{\gamma}_m {}^i\Psi^m({}^i\zeta)$ , ( $i = \eta, 1, \dots, N$ ) of the tangent section of principle bundle with the base  $\hat{G}_N = \hat{G}_\eta \oplus \hat{G}_{u_1} \oplus \dots \oplus \hat{G}_{u_N}$  should be promoted to the MW super operator vectors  $\hat{\tilde{\Phi}}_N({}^iZ^A)$  defined on the SOMM

$$\hat{S}G_N = \hat{S}G_\eta \oplus \hat{S}G_{u_1} \oplus \dots \oplus \hat{S}G_{u_N}, \quad (10.9)$$

where the following expansion holds

$$\hat{\tilde{\Phi}}_N({}^iZ^A) = {}^i\hat{\gamma}_M {}^i\tilde{\Phi}^M({}^iZ^A) = \hat{\Phi}({}^i\zeta) + \hat{\psi} + \hat{\bar{\psi}} \in \hat{S}G_N, \quad (10.10)$$

provided by the  ${}^i\hat{\gamma}_M = ({}^i\hat{\gamma}_m a, a^+)$ , and  ${}^i\partial_A = \frac{\partial}{\partial {}^iZ^A} = ({}^i\partial_m, \partial_\alpha, \bar{\partial}_{\dot{\alpha}})$ . In the case at hand the particles and sparticles are in the massless states since they are on the mass shell eq.(9.3). Such states can be analyzed from the light-like reference frame  $P_m = (p_\eta, 0, 0, p_u)$ , where the Casimirs are

$$P^2 = {}^iP_m {}^iP^m = 0, \quad C^2 = -2p_u^2(B_0 - B_3)^2 = -\frac{1}{2}p_u^2 \bar{Q}_2 Q_2 \bar{Q}_2 Q_2 = 0, \quad (10.11)$$

provided by  $\{Q_1, Q_1\} = 4p_u$ ,  $\{Q_2, Q_2\} = 0$ . Thus we can set  $Q_2 = 0$  in the operator sense, which yields only one pair of creation and annihilation operators

$$a = \frac{1}{2\sqrt{p_u}}Q_1, \quad a^+ = \frac{1}{2\sqrt{p_u}}Q_1. \quad (10.12)$$

The corresponding MW-SUSY generalized bilinear form can be readily computed to be in the form

$$\sum_{\lambda=\pm} \langle \tilde{\chi}_\lambda | \hat{\tilde{\Phi}}_N \hat{\tilde{\Phi}}_N | \tilde{\chi}_\lambda \rangle = \sum_{\lambda=\pm} \langle \chi_\lambda | \hat{\Phi}(\zeta) \hat{\Phi}(\zeta) | \chi_\lambda \rangle = -i\zeta^2 G_\zeta(0) = -i \left( \eta^2 G_\eta(0) - \sum_{i=1}^N u_i^2 G_{u_i}(0) \right), \quad (10.13)$$

which is required to be finite. The latter yields the major condition of the MW geometry realization

$$G_F(0) = G_\eta(0) = G_u(0), \quad u^2 G_{u_F}(0) \equiv \sum_{i=1}^N u_i^2 G_{u_i}(0), \quad (10.14)$$

or

$$m \equiv |p_u| \equiv \left( \sum_{i=1}^N p_{u_i}^2 \right)^{1/2} = |p_\eta|. \quad (10.15)$$

To produce the MW-SUSY generalization of the frame field defined on MW geometry, one must promote each left-handed fermionic components  $\chi_\eta(\eta)$  and  $\chi_u(u)$  in eq.(8.1) to corresponding chiral multiplets. Then, the total frame field itself will be a chiral superfield. Thus, the two irreps are important for further development of MSMSM. They are the chiral and vector multiplets of the MW-SUSY eq.(9.8), which will be discussed in the next section.

## 11 The supersymmetric frame field action defined on $SG_N$

The MW-SUSY (eq.(9.8)) generalization of the skeletonized frame field eq.(8.1), which is a major ingredient of microscopic structure of leptons and quarks, defined on the  $n = 1$  rigid supermanifold  $SG_N$  can be written in terms of chiral superfields

$$\tilde{\Phi}_{ch}(Z) = \tilde{\Phi}_\eta(Z_\eta) \tilde{\Phi}_{u_1}(Z_{u_1}) \cdots \tilde{\Phi}_{u_N}(Z_{u_N}), \quad (11.1)$$

where the generalized coordinates  $Z = ({}^i\zeta, \theta, \bar{\theta})$ ,  $Z_\eta = (\eta, \theta, \bar{\theta})$ ,  $Z_{u_1} = (u_1, \theta, \bar{\theta}), \dots$ ,  $Z_{u_N} = (u_N, \theta, \bar{\theta})$  parametrize the  $(N+1)(6+4)$ -dimensional coset space  $SG_N$ . According to the standard technique, the chiral irrep can be obtained from right covariant constrained general scalar superfield  $\tilde{\Phi}(Z)$ , which is not fully reducible  $\bar{D}_\alpha \tilde{\Phi}_{ch}(Z) = 0$ . To solve this constraint the scalar superfield is regarded as a function of new bosonic coordinates

$${}^i y^m = {}^i \zeta^m + i \theta {}^i \sigma^m \bar{\theta} \quad (11.2)$$

and  $\theta$ . Hence

$$\begin{aligned} \tilde{\Phi}_{ch}({}^i y^m, \theta) &= A({}^i y^m) + \sqrt{2} \theta \chi({}^i y^m) + \theta \theta F({}^i y^m) = A({}^i \zeta) + i \theta {}^i \sigma^m \bar{\theta} {}^i \partial_m A({}^i \zeta) + \\ &\frac{1}{4} \theta \theta \bar{\theta} \bar{\theta} \square A({}^i \zeta) + \sqrt{2} \theta \chi({}^i \zeta) - \frac{i}{\sqrt{2}} \theta \theta {}^i \partial_m \chi({}^i \zeta) {}^i \sigma^m \bar{\theta} + \theta \theta F({}^i \zeta), \end{aligned} \quad (11.3)$$

where  $\square = - {}^i \partial_m {}^i \partial^m$ ,  $\chi_\alpha({}^i \zeta)$  is the left-handed Weyl fermion of spin 1/2,  $A({}^i \zeta)$  is the scalar superpartner of spin 0, and  $F({}^i \zeta)$  is the complex scalar auxiliary field of spin 0. Thus,  $\tilde{\Phi}_{ch}$  has off-shell four real bosonic degrees of freedom  $n_B = 4$  and four real fermionic degrees of freedom  $n_F = 4$ . An infinitesimal  $n = 1$  supersymmetric transformation of the component fields are as follows:

$$\delta_\xi A = \sqrt{2} \xi \chi, \quad \delta_\xi \chi = i \sqrt{2} {}^i \sigma^m \bar{\xi} {}^i \partial_m A + \sqrt{2} \xi F, \quad \delta_\xi F = i \sqrt{2} \bar{\xi} {}^i \sigma^m {}^i \partial_m \chi. \quad (11.4)$$

Due to chain rule, the chiral superfield eq.(11.1) can be rewritten in terms of standard ingredients of chiral supermultiplet

$$\tilde{\Phi}_{ch}(Z) = \left( \chi({}^i \zeta), A({}^i \zeta) F({}^i \zeta) \right), \quad (11.5)$$

which have the following MW-component expansions:

$$\begin{aligned} \chi({}^i \zeta) &= \chi_\eta(\eta) \chi_{u_1}(u_1) \cdots \chi_{u_N}(u_N), \quad A({}^i \zeta) = A_\eta(\eta) A_{u_1}(u_1) \cdots A_{u_N}(u_N), \\ F({}^i \zeta) &= F_\eta(\eta) F_{u_1}(u_1) \cdots F_{u_N}(u_N), \end{aligned} \quad (11.6)$$

The local gauge transformations of chiral superfields read

$$\tilde{\Phi}_{ch}(Z) \rightarrow e^{\Lambda^a T^a} \tilde{\Phi}_{ch}(Z), \quad (11.7)$$

where, as usual,  $\Lambda$  is the chiral superfield satisfying the constraint  $\bar{D}\Lambda = 0$ . To make invariant the kinetic term  $\tilde{\Phi}_{ch}^+ \tilde{\Phi}_{ch}$  in the action of chiral superfield, one must introduce a vector superfield  $V = V^a T^a$ ,  $V = V^+$  to exponentiate the finite transformation

$$e^{gV} \rightarrow e^{i\Lambda^+} e^{gV} e^{-i\Lambda}, \quad (11.8)$$

where  $\Lambda \equiv \Lambda^a T^a$ . The generators  $T^a$  of a compact gauge group  $G$ , which will be specified in sec.12, have to commute with the supercharge operators, i.e., all members ( $\chi$ ,  $A$ ,  $F$ ) of chiral supermultiplet are in the same representation of the gauge group. Using a standard technique, without loss of generality one can decompose any vector superfield

$$V(\zeta, \theta, \bar{\theta}) = V_{WZ} + \tilde{\Phi}_{ch} + \tilde{\Phi}_{ch}^+, \quad (11.9)$$

and some of unphysical degrees of freedom then can be gauged away like the unitary gauge in ordinary field theory, such that in the Wess-Zumino gauge the vector superfield  $V_{WZ}$ , similar to the chiral multiplet, contains only equal number of bosonic and fermionic degrees of freedom  $n_B = n_F = 4$ : a gauge boson  ${}^i v_m({}^i \zeta)$  of spin 1 and mass dimension 1, a Weyl fermion (gaugino)  $\lambda$  of spin 1/2 and mass dimension 3/2, and a real scalar field  $D$  of spin 0 and mass dimension 2:  $V_{WZ} = ({}^i v_m({}^i \zeta), \lambda_\alpha({}^i \zeta), D({}^i \zeta), \cdot)$ . They all are Lie algebra valued fields, namely  ${}^i v_m = {}^i v_m^a T^a$ , etc. Hence,

$$V_{WZ} = -\theta {}^i \sigma^m \bar{\theta} {}^i v_m - i \bar{\theta} \bar{\theta} \lambda + i \theta \theta \bar{\theta} \bar{\lambda} + \frac{1}{2} \theta \theta \bar{\theta} \bar{\theta} D. \quad (11.10)$$

The SUSY transformations of the components read

$$\begin{aligned} \delta_\xi {}^i v_m^a &= -i \bar{\lambda}^a {}^i \bar{\sigma}^m \xi + i \xi {}^i \bar{\sigma}^m \lambda^a, & \delta_\xi \lambda^a &= i \xi D^a + \sigma^{mn} \xi F_{mn}^a, \\ \delta_\xi D^a &= -\xi {}^i \sigma^m {}^i D_m \bar{\lambda}^a - {}^i D_m \lambda^a {}^i \sigma^m \bar{\xi}, \end{aligned} \quad (11.11)$$

where  $\sigma^{mn} = \frac{1}{4} [{}^i \sigma^m, {}^i \sigma^n]$ , The Yang-Mills field strength of the vector bosons  $F_{mn}^a$  and covariant derivative are as follows:

$$\begin{aligned} F_{mn}^a &= {}^i \partial_n {}^i v_m^a - {}^i \partial_m {}^i v_n^a - g f^{abc} {}^i v_m^b {}^i v_n^c, \\ {}^i D_m \lambda^a &= {}^i \partial_m \lambda^a - g f^{abc} {}^i v_m^b \lambda^c, \end{aligned} \quad (11.12)$$

provided by the structure constants  $f^{abc}$  of the Lie algebra.

A gauge invariant, renormalizable, supersymmetric action of the gauge vector superfield reads

$$S_{gauge} = \frac{1}{4} \int d^6 \zeta \int d\theta^2 Tr W^\alpha W_\alpha, \quad (11.13)$$

where  $W^\alpha$  and  $\bar{W}_\alpha = (W_\alpha)^+$  are the left and right-handed spinor superfields. This is an off-shell irrep known as the curl multiplet or field strength multiplet containing  $4+1+3 = 8$  real components  $\lambda, D$  and  $F_{mn}$ . In the Wess-Zumino gauge the  $W^\alpha$  and  $\bar{W}_\alpha$  are written

$$W_\alpha \equiv -\frac{1}{4g} \bar{D} \bar{D} e^{-gV_{WZ}} D_\alpha e^{gV_{WZ}}, \quad \bar{W}_\alpha \equiv \frac{1}{4g} D D e^{gV_{WZ}} \bar{D}_{\dot{\alpha}} e^{-gV_{WZ}}. \quad (11.14)$$



A straightforward computation gives

$$W^\alpha(iy^m, \theta) = -i\lambda^\alpha(iy^m, \theta) + \theta^\alpha D(iy^m, \theta) - i\sigma_{mn}^\beta F_{mn}\theta_\beta + \theta\theta\sigma_{\alpha\dot{\alpha}}^m iD_m\bar{\lambda}^\alpha(iy^m), \quad (11.15)$$

and the action eq.(11.13) gives rise to

$$S_{gauge} = \int d^6\zeta \int d\theta^2 \left[ -\frac{1}{4} F_{mn}^a F^{mna} - i\bar{\lambda}^\alpha i\bar{\sigma}^m iD_m\lambda^\alpha + \frac{1}{2} D^\alpha D^\alpha \right]. \quad (11.16)$$

The  $F$ -component of a chiral superfield and the  $D$ -component of a vector superfield transform by a total derivative. Generic renormalizable action involving the chiral and vector gauge superfields [25-49] reads

$$S = \int d^6\zeta \int d^4\theta \tilde{\Phi}_{ch}^+ e^{gV} \tilde{\Phi}_{ch} + \left[ \int d^6\zeta \int d^2\theta \left( \frac{1}{4} Tr W^\alpha W_\alpha + P(\tilde{\Phi}_{ch}) \right) + \text{h.c.} \right]. \quad (11.17)$$

where  $P(\tilde{\Phi}_{ch})$  is the superpotential, which is the holomorphic function of  $\tilde{\Phi}_{ch}$  characterizing the interactions of chiral superfields of internal components

$$P(\tilde{\Phi}_{ch}) = P_{u_1}(\tilde{\Phi}_{ch}(u_1)) + \cdots + P_{u_N}(\tilde{\Phi}_{ch}(u_N)). \quad (11.18)$$

All the terms in eq.(11.17) fixed by symmetry expect for those in the superpotential. A gauge invariant, renormalizable Lagrangian containing the chiral multiplets ( $A^I, \chi^I, F^I$ ) explicitly labeled by the indices  $I, J, \dots$ , coupled to vector-multiplets stems from the eq.(11.17)

$$\begin{aligned} L(A^I, \chi^I, F^I, i v_m^a, \lambda^a, D^a) = & -iD_m A_I^* iD^m A^I - i\bar{\chi}_I i\bar{\sigma}^m iD_m \chi^I \\ & -\frac{1}{4} F_{mn}^a F^{mna} - i\bar{\lambda}^a i\bar{\sigma}^m iD_m \lambda^a - i\sqrt{2}g\bar{\lambda}^a \bar{\chi}_I T^{aI}_J A^J + i\sqrt{2}g A_J^* T^{aJ}_I \chi^I \lambda^a \\ & -\bar{F}^I F^I - \frac{g^2}{2} D^a D^a - g D^a A^{*I} T^a_{IJ} A^J - \frac{1}{2} P_{IJ} \chi^I \chi^J - \frac{1}{2} (P_{IJ})^* \bar{\chi}_I \bar{\chi}_J \\ & -F^I P_I - \bar{F}^I P_I^*, \end{aligned} \quad (11.19)$$

where

$$iD_m A^I = i\partial_m A^I + i v_m^a T^{aI}_J A^J, \quad iD_m \chi^I = i\partial_m \chi^I + i v_m^a T^{aI}_J \chi^J, \quad (11.20)$$

$P_I$  and  $P_{IJ}$  are the derivatives of a holomorphic function  $P(A)$ :

$$P_{IJ} = \frac{\partial^2}{\partial A^I \partial A^I} P(A), \quad P_I = \frac{\partial}{\partial A^I} P(A). \quad (11.21)$$

The only freedom in constructing the supersymmetric Lagrangian eq.(11.19) after choosing of superfields and gauge symmetries is to specify a superpotential  $P(\tilde{\Phi}_{ch})$ , which is not allowed to contain their complex conjugates  $\tilde{\Phi}_{ch}^+$ . Since the terms in the superpotential with more than 3 chiral superfields would yield non-renormalizable interactions in the Lagrangian, then it contains terms with 2 and 3 chiral superfields as in the Wess-Zumino simplest (sensible)  $n = 1$  SUSY toy model [25]. Furthermore, as an analytic function of

the superfields the superpotential is not allowed to contain derivative interactions. From the superpotential can be found both the scalar potential and Yukawa interactions of the fermions with the scalars

$$P(\tilde{\Phi}_{ch}) = \frac{1}{2} m \tilde{\Phi}_{ch}^2 + \frac{1}{3} Y \tilde{\Phi}_{ch}^3. \quad (11.22)$$

From the action eq.(11.17) one gets

$$\left[ \frac{1}{2} m \tilde{\Phi}_{ch}^2 + \frac{1}{3} Y \tilde{\Phi}_{ch}^3 \right]_{\theta\theta} = m A F + Y A^2 F. \quad (11.23)$$

Whence, the action eq.(11.17) contains no derivatives acting on the auxiliary field  $F$ , which can be eliminated by solving its equations of motion

$$\begin{aligned} \frac{\delta L}{\delta F_I} &= F_I^* - P_I = F_I^* - m_{IJ} A^J - Y_{IJK} A^J A^K = 0, \\ \frac{\delta L}{\delta F_I^*} &= F_I - P_I^* = F_I - m_{IJ} A^{*J} - Y_{IJK} A^{*J} A^{*K} = 0. \end{aligned} \quad (11.24)$$

The algebraic equation of motion for the other auxiliary field  $D^a$  can be obtained

$$\frac{\delta L}{\delta D^a} = D^a + g A^{*I} T_{IJ}^a A^{*J} = 0. \quad (11.25)$$

The equations (11.24) and (11.25) can be used to eliminate the auxiliary fields  $F^I$  and  $D^a$  from the Lagrangian eq.(11.19)

$$\begin{aligned} L(A^I, \chi^I, {}^i v_m^a, \lambda^a, F_I = P_I^*, D^a = -g A^{*I} T_{IJ}^a A^{*J}) &= -\frac{1}{4} F_{mn}^a F^{mna} \\ &- i \bar{\lambda}^a {}^i \bar{\sigma}^m {}^i D_m \lambda^a - {}^i D_m A_I^* {}^i D^m A^I - i \bar{\chi}_I {}^i \bar{\sigma}^m {}^i D_m \chi^I + i \sqrt{2} g (A_J^* T_I^{aJ} \chi^I \lambda^a \\ &- \bar{\lambda}^a \bar{\chi}_I T_J^{aI} A^J) - \frac{1}{2} P_{IJ} \chi^I \chi^J - \frac{1}{2} (P_{IJ})^* \bar{\chi}_I \bar{\chi}_J - V(A, A^*), \end{aligned} \quad (11.26)$$

where the positive semi-definite scalar potential  $V(A, A^*)$  is extracted:

$$\begin{aligned} V(A, A^*) &= \left( F^I F^{*I} + \frac{1}{2} D^a D^a \right) \frac{\delta L}{\delta F} = 0, \quad \frac{\delta L}{\delta D^a} = 0 \\ P_I P_I^* + \frac{1}{2} g^2 (A^{*I} T_{IJ}^a A^J) (A^{*I} T_{IJ}^a A^J) &\geq 0. \end{aligned} \quad (11.27)$$

We purposely expose the explicit mass term and Yukawa coupling in the superpotential eq.(11.22) in order to emphasize the most remarkable feature of SUSY theory that such choices are fixed by supersymmetry and ensured that all quadratic divergences cancel between bosonic and fermionic loops. Certainly, from the eq.(11.26) can be seen that like the Wess-Zumino simplest toy model the fields interact via Yuakawa and scalar couplings, where the quadratic divergences exactly cancel due to the supersymmetric relations  $Y_F = Y$ , and  $Y_B = Y^* Y$ . But the values of the masses and Yukawa couplings in eq.(11.24) still are free parameters of the theory at hand, which are implemented as much as they needed to be further constrained by the appropriate choice of the concrete MW-superpotential eq.(11.18) inserting the nonlinear interactions of internal components of the MW frame

field. The latter will be adopted to build up the SUSY extension of microscopic MW structure of all the particles in MSM. Here we shall attempt to amplify and substantiate the assertions made in the MSM and further expose via MSMSM. In pursuing this aim we are once again led to the principal point of drastic change of the standard SUSY scheme to specialize, hereinafter, the superpotential eq.(11.18) to be in the form

$$P(Q) = P_Q(A_Q) + P_W(A_W) \quad (11.28)$$

such that

$$|F|^2 = |F_Q|^2 + |F_W|^2 = \left| \frac{\partial P_Q}{\partial A_Q} \right| + \left| \frac{\partial P_Q}{\partial A_W} \right|, \quad (11.29)$$

provided by the scalars

$$\left| \frac{\partial P_Q}{\partial A_Q} \right| \equiv L_Q = \frac{1}{4} \lambda_Q \left( \frac{J_L}{Q} \frac{J_R^+}{Q} + \frac{J_R}{Q} \frac{J_L^+}{Q} \right), \quad \left| \frac{\partial P_Q}{\partial A_W} \right| \equiv L_W = \frac{1}{2} \lambda_W S_W S_W^+. \quad (11.30)$$

Here we admit that the MW-index ( $i$ ) will be running through  $i = Q, W$  specifying the internal worlds formally denoting  $Q$ -world of electric charge and the  $W$ -world of weak interactions [2](subsec.12.2 and 12.3). Also adopt the conventions of [2]

$$\begin{aligned} \frac{J}{Q_{L,R}} &= V \mp A, \quad V_Q^m = \bar{\Psi}_Q \gamma^m \psi_Q, \quad (V_Q^m)^+ = V_Q^m = \bar{\Psi}_Q \gamma_m \psi_Q, \\ A_Q^m &= \bar{\Psi}_Q \gamma^m \gamma^5 \psi_Q, \quad (A_Q^m)^+ = A_Q^m = \bar{\Psi}_Q \gamma_m \gamma^5 \psi_Q, \quad S_W = \bar{\Psi}_W \psi_W, \\ m &\equiv (\lambda, \delta) \quad \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3, \quad \lambda = \pm, \quad \delta = 1, 2, 3 \end{aligned} \quad (11.31)$$

$\gamma^m$  are given in App.A,  $\gamma^\mu$  and  $\gamma^5$  are Dirac matrices,  $\psi_D$  is the Dirac spinor in the Weyl basis containing a left-handed Weyl spinor  $\chi_\alpha$  and a right-handed Weyl spinor  $P_L \psi_D = \begin{pmatrix} \chi_\alpha \\ 0 \end{pmatrix}$ ,  $P_R \psi_D = \begin{pmatrix} 0 \\ \psi_{\dot{\alpha}} \end{pmatrix}$ . Using the Fiertz identities one gets

$$L_Q = \frac{1}{2} \lambda_Q (V_Q V_Q^+ - A_Q A_Q^+) = -\lambda_Q (S_Q S_Q^+ - p_Q p_Q^+), \quad (11.32)$$

where  $S_Q = \bar{\psi}_D \psi_D$  and  $p_Q = \bar{\psi}_D \gamma^5 \psi_D$ . Thus, despite the fact that in conventional SUSY theories the superpotential gives rise the fermion masses and Yukawa couplings, in suggested framework, in accordance with eq.(11.30), such terms are substituted by a new ones using a following set of constraints

$$\begin{aligned} F_Q^* &= -\frac{\partial P_Q}{\partial A_Q^I} \equiv 2i(\lambda_Q)^{1/2} \left( \chi_Q \psi_Q \right) = m_{IJ}^Q A_Q^J + Y_{IJK}^Q A_Q^J A_Q^K, \\ F_W^* &= -\frac{\partial P_W}{\partial A_W^I} \equiv i\left(\frac{\lambda_W}{2}\right)^{1/2} \left( \chi_W \psi_W + \bar{\chi}_W \bar{\psi}_W \right) = m_{IJ}^W A_W^J + Y_{IJK}^W A_W^J A_W^K, \\ P_{IJ}^Q &\equiv \frac{\partial}{\partial A_Q^I} \left[ -2i(\lambda_Q)^{1/2} \left( \chi_Q \psi_Q \right) \right] = m_{IJ}^Q + 2 Y_{IJK}^Q A_Q^K, \\ P_{IJ}^W &\equiv \frac{\partial}{\partial A_W^I} \left[ -i\left(\frac{\lambda_W}{2}\right)^{1/2} \left( \chi_W \psi_W + \bar{\chi}_W \bar{\psi}_W \right) \right] = m_{IJ}^W + 2 Y_{IJK}^W A_W^K, \end{aligned} \quad (11.33)$$

where, as usual, one abbreviates expressions with two-spinor fields by suppressing undotted indices contracted like  $^\alpha_\alpha$ , and dotted indices contracted like  $_{\dot{\alpha}}^{\dot{\alpha}}$ . While, the positive semidefinite scalar potential eq.(11.27) gives rise to

$$V(A, A^*) = L_Q + L_W + \frac{1}{2} g^2 \left( A^{*I} T_{IJ}^a A^J \right)^2 \geq 0. \quad (11.34)$$

Furthermore, putting it all together and inserting eq.(11.34) into eq.(11.26) with  $P_{IJ} \equiv 0$  the Lagrangian of supersymmetric frame fields reduces to one contained only the MW internal components of particles and their SUSY partners such that all of them are massless:

$$\begin{aligned}
L(A^I, \chi^I, {}^i v_m^a, \lambda^a) = & -\frac{1}{4} F_{mn}^a F^{mna} - i \bar{\lambda}^a {}^i \bar{\sigma}^m {}^i D_m \lambda^a - {}^i D_m A_I^* {}^i D^m A^I \\
& - i \bar{\chi}_I {}^i \bar{\sigma}^m {}^i D_m \chi^I + i\sqrt{2} g \left( A_J^* T^{aJ}_I \chi^I \lambda^a - \bar{\lambda}^a \bar{\chi}_I T^{aI}_J A^J \right) + 4 \lambda_Q \left( \chi_Q \psi_Q \right) \left( \bar{\chi}_Q \bar{\psi}_Q \right) \\
& - \frac{\lambda_W}{2} \left( \chi_W \psi_W + \bar{\chi}_W \bar{\psi}_W \right)^2 - \frac{1}{2} g^2 \left( A^{*I} T_{IJ}^a A^J \right)^2,
\end{aligned} \tag{11.35}$$

where all the fields  $A^I, \chi^I, {}^i v_m^a, \lambda^a$  imply the MW-expansion.

## 12 The brief outline of theoretical issues in the MSM

We now have all the tools we need to build up the MSMSM. For our immediate purpose, however, to provide sufficient background let us recapitulate. The following subsections contain concisely some of the necessary knowledge of generic on MSM, but for more details we refer to [2]. Within the MSM the possible elementary particles are thought to be composite dynamical systems in analogy to quantum mechanical stationary states of compound atom, i.e., the systems arisen from the primary fundamental principle. But, now a dynamical treatment built up on the MW geometry is quite different and more amenable to qualitative understanding.

### 12.1 The MW-structure of the particles

We start by considering the MW-structure of all the collection of matter fields  $\Psi(\zeta)$  with nontrivial MW internal structure  $\Psi(\zeta) = \psi(\eta) {}^1 \psi(\theta_1) \cdots {}^N \psi(\theta_N)$ , where the component  ${}^i \psi(\theta_i)$  is made of product of some subquarks and antisubquarks  ${}^i \psi(\theta) = {}^i \psi(\{^i q\}, \{^i \bar{q}\})$ . We admit that the MW index ( $i$ ) will be running only through  $i = Q, W, B, s, c, b, t$  specifying the internal worlds formally taken to denote in following nomenclature: Q-world of electric charge; W-world of weak interactions; B-baryonic world of strong interactions; the s,c,b,t are the worlds of strangeness, charm, bottom and top. Also, we admit that the distortion rotations in the worlds Q,W and B are local depending only of the  $\eta$ -coordinates  ${}^i \theta_{\pm k}(\eta)$  and they are global in the flavour worlds s,c,b,t. We note that due to concrete symmetries of internal worlds, the MW- structure of particles will come into being if the following major conditions hold, namely, besides the MW geometry realization requirement eq.(3.1.4) a condition of MW connections must be held too, which will be discussed in the next subsection. We assign to each distortion rotation mode in the three dimensional spaces  $R_{u_i}^3$  and  $R_{u_i}^3$  a scale 1/3, namely each of the subquarks associated with the rotations around the axes of given world carries the corresponding charge in the scale 1/3 ; antisubquark carries respectively the  $(-1/3)$  charge. In the case of the worlds C=s,c,b,t, where distortion rotations are global and diagonal with respect to axes 1,2,3, the physical system of corresponding subquarks is invariant under the global transformations  $f_C^{(3)}(\theta^c)$  of the global unitary group  $diag(SU_3^c)$  parametrized by the three angles  $\theta_1^c, \theta_2^c$

and  $\theta_3^c$ . While  $f_C^{(3)} (f_C^{(3)})^+ = 1$ ,  $\|f_C^{(3)}\| = 1$ . That is  $\theta_1^c + \theta_2^c + \theta_3^c = 0$ . We explore the simplest possibility  $\theta^c \equiv \theta_1^c = \theta_2^c$ , then one gets

$$f_C^{(3)} = \exp \left( -i \frac{\lambda_8}{\sqrt{3}} \theta^c \right) = e^{-iY^c \theta^c}, \quad (12.1.1)$$

provided by the operator of hypercharge  $Y^c$  of diagonal group  $diag(SU_i^c)$ . If all the internal worlds are involved, then  $Y^c = s + c + b + t$ . The conservation of each rotation mode in Q- and B- worlds, where the distortion rotations are local, means that corresponding subquarks carry respectively the conserved charges Q and B in the scale 1/3, and antisubquarks - (-1/3) charges. It can be provided by including the matrix  $\lambda_8$  as the generator with the others in the symmetries of corresponding worlds (Q, B), and expressed in the invariance of the system of corresponding subquarks under the transformations of these symmetries. The incompatibility relations eq.(2.5.2) for global distortion rotations in the worlds C=s,c,b,t reduced to

$$f_{11}^c f_{22}^c = \bar{f}_{33}^c, \quad f_{22}^c f_{33}^c = \bar{f}_{11}^c, \quad f_{33}^c f_{11}^c = \bar{f}_{22}^c,$$

where  $\|f_C^{(3)}\| = f_{11}^c f_{22}^c f_{33}^c = 1$ ,  $f_{ii}^c \bar{f}_{ii}^c = 1$  for  $i = 1, 2, 3$ . It means that two sub-colour singlets are available:  $(q\bar{q})_i^c = inv$ ,  $(q_1 q_2 q_3)^c = inv$ , carrying respectively the charges  $C_{(q\bar{q})_i^c} = 0$   $C_{(q_1 q_2 q_3)^c} = 1$ . We make use of convention  $(q\bar{q})_i^c \equiv {}^c q_i {}^c \bar{q}_i$ ,  $(q_1 q_2 q_3)^c \equiv {}^c q_1 {}^c q_2 {}^c q_3$ . Including the baryonic charge into strong hypercharge

$$Y = B + s + c + b + t, \quad (12.1.2)$$

we conclude that the hypercharge Y is a sum of all conserved rotation modes in the internal worlds B,s,c,b,t involved in the MW geometry realization condition eq.(3.1.4).

## 12.2 Realization of Q-world and Gell-Mann-Nishijima relation

The symmetry of Q-world of electric charge, assumed to be a local unitary symmetry  $diag(SU^{loc}(3))$ , is diagonal with respect to axes 1,2,3. The unitary unimodular matrix  $f_Q^{(3)}$  of local distortion rotations takes the form

$$f_Q^{(3)} = e^{-i\lambda_Q \theta_Q},$$

where  $f_Q^{(3)} (f_Q^{(3)})^+ = 1$ ,  $\|f_Q^{(3)}\| = 1$ , provided  $\theta_1 + \theta_2 + \theta_3 = 0$ . Taking into account the scale of rotation mode, in other than eq.(12.1.1) simple case  $\theta_2 = \theta_3 = -\frac{1}{3}\theta_Q$  it follows that  $\theta_1 = \frac{2}{3}\theta_Q$ . The matrix  $\lambda_Q$  may be written  $\lambda_Q = \frac{1}{2}\lambda_3 + \frac{1}{2\sqrt{3}}\lambda_8$ . Making use of the corresponding operators of the group  $SU(3)$  we arrive at Gell-Mann-Nishijima relation

$$Q = T_3 + \frac{1}{2}Y, \quad (12.2.1)$$

where  $Q = \lambda_Q$  is the generator of electric charge,  $T_3 = \frac{1}{2}\lambda_3$  is the third component of isospin  $\vec{T}$ , and  $Y = \frac{1}{\sqrt{3}}\lambda_8$  is the hypercharge. The eigenvalues of these operators

will be defined later on by considering the symmetries and microscopic structures of fundamental fields. We think of operators  $T_3$  and  $Y$  as the MW connection charges and of relation eq.(12.2.1) as the condition of the realization of the MW connections. Thus, during realization of MW- structure the symmetries of corresponding internal worlds must be unified into more higher symmetry including also the  $\lambda_3$  and  $\lambda_8$ . Meanwhile, the realization conditions of the MW- structure are embodied in eq.(3.1.4) and eq.(12.2.1), provided by the conservation law of each rotational mode in the corresponding internal worlds involved. For example, in the case of quarks (see eq.(12.5.1)) the eq.(3.1.4) reads

$$\sum_{i=B,s,c,b,t} \omega_i G_{iF}^\theta(0) = G_{\eta F}^\theta(0), \quad (12.2.2)$$

and according to eq.(12.1.5), the Gell-Mann-Nishijima relation is written down

$$Q = T_3 + \frac{1}{2}(B + s + c + b + t). \quad (12.2.3)$$

In the case of leptons (see eq.(12.4.1)), the realization conditions reduced to the following:

$$G_{Q_F}^\theta(0) = G_{\eta_F}^\theta(0), \quad u \equiv u_Q, \quad (i \equiv Q) \quad (12.2.4)$$

and

$$Q = T_3^w + \frac{1}{2}Y^w, \quad (12.2.5)$$

where  $T_3^w$  and  $Y^w$  are respectively the operators of third component of weak isospin  $\vec{T}^w$  and weak hypercharge (subsec.12.3, 12.8). The incompatibility relations eq.(2.5.2) lead to

$$f_{11}^Q f_{22}^Q = \bar{f}_{33}^Q, \quad f_{22}^Q f_{33}^Q = \bar{f}_{11}^Q, \quad f_{33}^Q f_{11}^Q = \bar{f}_{22}^Q,$$

where  $f_{ii}^Q \bar{f}_{ii}^Q = 1$ , for  $i = 1, 2, 3$ ,  $\|f_Q^{(3)}\| = f_{11}^Q f_{22}^Q f_{33}^Q = 1$ . This in turn suggests two subcolour singlets  $(q\bar{q})_i^Q = inv$ ,  $(q_1 q_2 q_3)^Q = inv$ , with the electric charges  $Q_{(q\bar{q})_i^Q} = 0$ ,  $Q_{(q_1 q_2 q_3)^Q} = 1$ , respectively. The singlets  $(q\bar{q})_i^Q$  for given  $i$  allow us to think of the  $(q\bar{q})^Q$  system as the mixed ensemble, such that a fraction of the members with relative population  $L_1$  are characterized by the  $(q_1)^Q$ , some other fraction with relative population  $L_2$ , by  $(q_2)^Q$ , and so on. Namely, the  $(q\bar{q})^Q$  ensemble can be regarded as a mixture of pure ensembles. The fractional populations are constrained to satisfy the normalization condition

$$\sum_i L_i = 1, \quad L_i \equiv (q\bar{q})_i^Q / (q\bar{q})^Q. \quad (12.2.6)$$

The  $L_i$  also imply the orthogonality condition ensued from the symmetry of the  $Q$ -world

$$\langle L_i, L_j \rangle = 0 \quad \text{if } i \neq j. \quad (12.2.7)$$

This prompts us to define the usual quantum mechanical density operator

$$\rho_1^Q = \sum_i L_i (q_i)^Q (q_i)^{Q+}, \quad \text{tr}(\rho_1^Q) = 1. \quad (12.2.8)$$

The eq.(12.2.6) suggests another singlets as well

$$(q_1 q_2 q_3)_i^Q \equiv L_i (q_1 q_2 q_3)^Q, \quad (12.2.9)$$

which will be used to build up the MW-structures of the leptons.

### 12.3 The symmetries of the W-,B-, and global worlds

- The W-world

It will be seen in subsec.12.8 that the symmetry of W-world of weak interactions is  $SU^{loc}(2)_L \otimes U^{loc}(1)_Y$  invoking local group of weak hypercharge  $Y^w$  ( $U^{loc}(1)_Y$ ). However, for the present it is worthwhile to restrict oneself by admitting that the symmetry of W-world is simply expressed by the group of weak isospin  $SU^{loc}(2)$ , i.e., from the very first we consider the case of two dimensional distortion transformations through the angles  $\theta_{\pm}$  around two arbitrary axes in the W-world. In accordance with the results of subsec.3.2, the fields of subquarks and antishquarks will come in doublets, which form the basis for fundamental representation of weak isospin group  $SU^{loc}(2)$  often called a “custodial” symmetry [5,20,126]. The doublet states are complex linear combinations of up and down states of weak isotopic spin. Three possible doublets of six subquark states are  $\begin{pmatrix} q_1 \\ q_2 \end{pmatrix}^W$ ,  $\begin{pmatrix} q_2 \\ q_3 \end{pmatrix}^W$ ,  $\begin{pmatrix} q_3 \\ q_1 \end{pmatrix}^W$ .

- The B-world

The B-world is responsible for strong interactions. The internal symmetry group is  $SU_c^{loc}(3)$  enabling to introduce gauge theory in subcolour space with subcolour charges as exactly conserved quantities (sec.3 in [1]). The local distortion transformations implemented on the subquarks  $(q_i)^B$ ,  $i = 1, 2, 3$  through a  $SU_c^{loc}(3)$  rotation matrix  $U$  in the fundamental representation. Taking into account a conservation of rotation mode, each subquark carries (1/3) baryonic charge, while the antishquark carries the (−1/3) baryonic charge.

- The global worlds

We adopt a simplified view-point on the field component  $(q_f^c)$  ( $f=u,d,s,c,b,t$ ) associated with the global distortion rotations in the given worlds s,c,b,t, such that they have following microscopic structure with corresponding global charges:

$$\begin{aligned} q_u^c = q_d^c = 1, \quad \bar{q}_s^c = (\bar{q}_1^c \bar{q}_2^c \bar{q}_3^c)^s, \quad s = -1; \quad q_c^c = (q_1^c q_2^c q_3^c)^c, \quad c = 1; \\ \bar{q}_b^c = (\bar{q}_1^c \bar{q}_2^c \bar{q}_3^c)^b, \quad b = -1; \quad q_t^c = (q_1^c q_2^c q_3^c)^t, \quad t = 1. \end{aligned} \quad (12.3.1)$$

To realize the MW-structure the global symmetries of internal worlds have unified into more higher symmetry including the generators  $\lambda_3$  and  $\lambda_8$  (subsec.12.2). This global group is the flavour group  $SU_f(6)$  unifying all the symmetries  $SU_i^c$  of the worlds Q,B,s,c,b,t:  $SU_f(6) \supset SU_f(2) \otimes SU_B^c \otimes SU_s^c \otimes SU_c^c \otimes SU_b^c \otimes SU_t^c$ .

### 12.4 The microscopic structure of leptons

After a quantitative discussion of the properties of symmetries of internal worlds, below we will attempt to show how the known fermion fields of leptons and quarks fit into this scheme. In this section we start with the leptons. Taking into account the eq.(12.1.1) and eq.(12.2.4), we may consider six possible lepton fields forming three doublets of lepton

generations  $\begin{pmatrix} \nu_e \\ e \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$ , where

$$\begin{cases} \nu_e \equiv \psi_{\nu_e}(\eta) (q_1 \bar{q}_1)^Q (q_1)^w = L_e \psi_{\nu_e}(\eta) (q \bar{q})^Q (q_1)^w, \\ e \equiv \psi_e(\eta) (\bar{q}_1 q_2 q_3)_1^Q (q_2)^w = L_e \psi_e(\eta) (\bar{q}_1 q_2 q_3)^Q (q_2)^w, \\ \nu_\mu \equiv \psi_{\nu_\mu}(\eta) (q_2 \bar{q}_2)^Q (q_2)^w = L_\mu \psi_{\nu_\mu}(\eta) (q \bar{q})^Q (q_2)^w, \\ \mu \equiv \psi_\mu(\eta) (\bar{q}_1 q_2 q_3)_2^Q (q_3)^w = L_\mu \psi_\mu(\eta) (\bar{q}_1 q_2 q_3)^Q (q_3)^w, \\ \nu_\tau \equiv \psi_{\nu_\tau}(\eta) (q_3 \bar{q}_3)^Q (q_3)^w = L_\tau \psi_{\nu_\tau}(\eta) (q \bar{q})^Q (q_3)^w, \\ \tau \equiv \psi_\tau(\eta) (\bar{q}_1 q_2 q_3)_3^Q (q_1)^w = L_\tau \psi_\tau(\eta) (\bar{q}_1 q_2 q_3)^Q (q_1)^w \end{cases} \quad (12.4.1)$$

Here  $e, \mu, \tau$  are the electron, the muon and the tau meson,  $\nu_e, \nu_\mu, \nu_\tau$  are corresponding neutrinos,  $L_e \equiv L_1, L_\mu \equiv L_2, L_\tau \equiv L_3$ , are leptonic charges. The leptons carry leptonic charges as follows:  $L_e : (e, \nu_e)$ ,  $L_\mu : (\mu, \nu_\mu)$  and  $L_\tau : (\tau, \nu_\tau)$ , which are conserved in all interactions. The leptons carry also the weak isospins:  $T_3^w = \frac{1}{2}$  for  $\nu_e, \nu_\mu, \nu_\tau$ ; and  $T_3^w = -\frac{1}{2}$  for  $e, \mu, \tau$ , respectively, and following electric charges:  $Q_{\nu_e} = Q_{\nu_\mu} = Q_{\nu_\tau} = 0$ ,  $Q_e = Q_\mu = Q_\tau = -1$ . The Q-components  $\psi(u_Q)$  of lepton fields eq.(12.4.1) are made of singlet combinations of subquarks in Q-world. They imply subcolour confinement eq.(12.2.4). Then, the MW geometry realization condition is already satisfied and leptons may emerge in geometry in free combinations without any constraint. Thus, in suggested scheme there are only three possible generations of six leptons with integer electric and leptonic charges being free of confinement.

## 12.5 The microscopic structure of quarks

The only possible MW- structures of 18 quark fields read

$$\begin{cases} u_i \equiv \psi_u(\eta) (q_2 q_3)^Q (q_1)^w (q_i^B), \\ d_i \equiv \psi_d(\eta) (\bar{q}_1)^Q (q_2)^w (q_i^B), \\ t_i \equiv \psi_t(\eta) (q_1 q_2)^Q (q_3)^w (q_i^B) (q_t^c), \\ b_i \equiv \psi_b(\eta) (\bar{q}_3)^Q (q_1)^w (q_i^B) (\bar{q}_b^c), \end{cases} \quad \begin{cases} c_i \equiv \psi_c(\eta) (q_3 q_1)^Q (q_2)^w (q_i^B) (q_c^c), \\ s_i \equiv \psi_s(\eta) (\bar{q}_2)^Q (q_3)^w (q_i^B) (\bar{q}_s^c), \end{cases} \quad (12.5.1)$$

where the subcolour index ( $i$ ) runs through  $i = 1, 2, 3$ , the  $(q_f^c)$  are given in eq.(12.1.3). Henceforth the subcolour index will be left implicit, but always a summation must be extended over all subcolours in B-world. These fields form three possible doublets of weak isospin in the W-world  $\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}$ . The quark flavour mixing and similar issues are left for discussion in subsec.17.1. The corresponding electric charges of quarks read  $Q_u = Q_c = Q_t = \frac{2}{3}$ ,  $Q_d = Q_s = Q_b = -\frac{1}{3}$ , in agreement with the rules governing the MW connections eq.(12.2.3), where the electric charge difference of up and down quarks implies  $\Delta Q = \Delta T_3^w = 1$ . The explicit form of structure of  $(q_f^c)$  are given in the next section. Note that all components of  $(q_f^c)$  are made of singlet combinations



(eq.(12.1.1)) of global subquarks in corresponding internal worlds, i.e., they obey the sub-colour confinement condition. According to eq.(12.2.2), this condition for B-world still remains to be satisfied. Therefore the total quark fields obey the confinement. Thus, three quark generations of six possible quark fields exist. They carry fractional electric and baryonic charges and imply a confinement. Although within considered schemes the subquarks are defined on the internal worlds, however the resulting  $\eta$ -components, which we are going to deal with to describe the leptons and quarks defined on the spacetime continuum, are affected by them. Actually, as it is seen in subsec.2.4 the rotation through the angle  $\theta_{+k}$  yields a total subquark field

$$q_k(\theta) = \Psi(\theta_{+k}) = \psi_{\eta}^0 \psi_u(\theta_{+k})$$

where  $\psi_{\eta}^0$  is the plane wave defined on  $G_{\eta}$ . Hence, one gets

$$q_k(\theta(\eta)) = \psi_{\eta}^0 q_u(\theta(\eta)) = q_u(\theta(\eta)) \psi_u^0, \quad q_k(\theta(\eta)) \equiv f_{(+)}(\theta_{+k}(\eta)) \psi_{\eta}^0,$$

where  $\psi_u^0$  is a plane wave defined on  $G_u$ . The  $q_k(\theta(\eta))$  can be considered as the subquark field defined on the flat manifold  $G_{\eta}$  with the same quantum numbers of  $q_u(\theta(\eta))$ . Thus, instead of the eq.(12.4.1) and eq.(12.5.1) we may consider on equal footing only the resulting  $\eta$ -components of leptons and quarks implying the given same structures. This enables to pass back to the Minkowski spacetime continuum  $G_{\eta} \rightarrow M_4$  (subsec.2.1).

## 12.6 The particle frame field

All the fields including the leptons eq.(12.4.1) and quarks eq.(12.5.1), along with the spacetime components have also MW components made of the various subquarks defined on the corresponding internal worlds:

$$\Psi(\theta) = \psi_{\eta}(\eta) \psi_Q(\theta_Q) \psi_W(\theta_W) \psi_B(\theta_B) \psi_C(\theta^c). \quad (12.6.1)$$

We assume that this field has arisen from the frame field with the same components defined on  $G_N$  in the lowest state ( $s_0$ )

$$\Psi(0) = \psi_{\eta}(\eta) \psi_Q(0) \psi_W(0) \psi_B(0) \psi_C(0) \quad (12.6.2)$$

which serves as the ready made frame into which the distorted ordinary structures of the same species should be involved. The components  $\psi_Q(\theta_Q), \psi_W(\theta_W), \psi_B(\theta_B)$  are primary massless bare Fermi fields, Let us now extract from the MW-SUSY Lagrangian eq.(11.35) the piece containing only  $i = \eta, Q, W$  fermionic components of the particle frame fields. To keep resemblance with [2] it is convenient to rewrite them in Dirac spinor terms suppressing the subscript  $D$  and superscript  $I$  at  $\psi_D^I$ . The resulting Lagrangian of the frame field with nonlinear fermion interactions of the internal components looks like Heisenberg theory and reads

$$L(D) = L_{\eta}(\eta) - L_Q(Q) - L_W(W), \quad (12.6.3)$$

where

$$\begin{aligned} L_{\eta\eta}(D) &= L'_{\eta 0}{}^{(0)}(D) - \frac{1}{2}Tr(\mathbf{B}\bar{\mathbf{B}}_{\eta\eta}), & L_Q(D) &= L'_{Q 0}{}^{(0)}(D) - L_Q - \frac{1}{2}Tr(\mathbf{B}\bar{\mathbf{B}}_Q), \\ L_W(D) &= L'_{W 0}{}^{(0)}(D) - L_W - \frac{1}{2}Tr(\mathbf{B}\bar{\mathbf{B}}_W). \end{aligned} \quad (12.6.4)$$

Here

$$\begin{aligned} L'_{\eta 0}{}^{(0)} &= \frac{i}{2}\{\bar{\Psi}\gamma D_{\eta}\Psi - \bar{\Psi}\gamma \overleftarrow{D}_{\eta}\Psi\} = \psi_u^+ L_{\eta}^{(0)} \psi_u, \\ L'_{u 0}{}^{(0)} &= \frac{i}{2}\{\bar{\Psi}\gamma D_u\Psi - \bar{\Psi}\gamma \overleftarrow{D}_u\Psi\} = \psi_{\eta}^+ L_u^{(0)} \psi_{\eta}, \end{aligned} \quad (12.6.5)$$

and

$$L_0^{(0)} = \frac{i}{2}\{\bar{\Psi}\gamma D_{\eta}\psi_{\eta} - \bar{\Psi}\gamma \overleftarrow{D}_{\eta}\psi_{\eta}\}, \quad L_u^{(0)} = \frac{i}{2}\{\bar{\Psi}_u\gamma D_u\psi_u - \bar{\Psi}_u\gamma \overleftarrow{D}_u\psi_u\}. \quad (12.6.6)$$

The Lagrangian eq.(12.6.3) has the global  $\gamma_5$  and local gauge symmetries. We consider only  $\gamma_5$  symmetry in Q-world, namely  $\mathbf{B}_Q \equiv 0$ . According to the OMM formalism, it is the most important to fix the mass shell of the stable MW- structure eq.(3.5). Thus, in the first we must take the variation of the Lagrangian eq.(12.6.3) with respect to frame field eq.(12.6.2), then switch on nonlinear fermion interactions of the components. In other words we shall take the variation of the Lagrangian eq.(12.7.3) with respect to these components only on the fixed mass shell. The equations of free field ( $\mathbf{B} = 0$ ) of the MW- structure follow at once, which can be written in terms of separate equations for the massless bare components  $\psi_{\eta}$ ,  $\psi_Q$  and  $\psi_W$ :

$$\gamma p_{\eta} \psi_{\eta} = i \gamma \partial_{\eta} \psi_{\eta} = 0, \quad \gamma p_Q \psi_Q = i \gamma \partial_Q \psi_Q = 0, \quad \gamma p_W \psi_W = i \gamma \partial_W \psi_W = 0. \quad (12.6.7)$$

The important feature is that these equations respect the simultaneous substitution  $\psi_Q^{(0)} \rightarrow \psi_Q^{(m)}$  and  $\psi_{\eta}^{(0)} \rightarrow \psi_{\eta}^{(m)}$ , where  $\psi_Q^{(0)}$  and  $\psi_{\eta}^{(m)}$  are respectively the massless and massive  $Q$ ,  $\eta$ -component fields. Then, in free state the massless field components  $\psi_{\eta}$ ,  $\psi_Q$  and  $\psi_W$  are independent the Lagrangian

$$L_0'^{(0)} = \psi_u^+ L_{\eta}^{(0)} \psi_u - \psi_{\eta}^+ L_u^{(0)} \psi_{\eta} = \psi_u^+ L_{\eta}^{(0)} \psi_u - \psi_{\eta}^+ (L_Q^{(0)} + L_W^{(0)}) \psi_{\eta} \quad (12.6.8)$$

reduces to the following:

$$L_0'^{(0)} = L_{\eta}^{(0)} - L_u^{(0)} = L_{\eta}^{(0)} - L_Q^{(0)} - L_W^{(0)}. \quad (12.6.9)$$

Thus, we shall implement our scheme as follows: starting with the reduced Lagrangian  $L_0'^{(0)}$  of free field we shall switch on nonlinear fermion interactions of the components. After a generation of nonzero mass of  $\psi_Q$  component in Q-world (next subsec.) shall look for the corresponding corrections via the eq.(12.6.5) to the reduced Lagrangian eq.(12.6.9). These corrections mean only the interaction between the components, and do not imply at all the mass acquiring process for the  $\eta$ -component.

## 12.7 Generation of mass of fermions in Q-world

We apply now a well known Nambu-Jona-Lasinio model [95] to generate a fermion mass in the Q-world and start from the chirality invariant total Lagrangian of the field  $\psi_Q$ :

$L = L_Q^{(0)} - L_{Q_I}$ , where a primary field  $\psi$  is the massless bare spinor implying  $\gamma_5$  invariance.

However, due to interaction the rearrangement of vacuum state has caused a generation of nonzero mass of fermion such like to appearance of energy gap in superconductor [96-98] and [99, 100]. After a vacuum rearrangement the total Lagrangian of initial massless bare field  $\psi_Q^0$  gives rise to corresponding Lagrangian  $L_Q^{(m)}$  of massive field  $\psi_Q^{(m)}$ :

$L = L_Q^{(0)} - L_{Q_I} = L_Q^{(m)}$  describing Dirac particle  $(\gamma p_Q - \Sigma_Q)\psi_Q^{(m)} = 0$ , where  $\Sigma_Q$  is the

self-energy operator. In lowest order  $\Sigma_Q = m_Q \ll \lambda^{-1/2}$ . Within the refined theory of superconductivity, the collective excitations of quasi-particle pairs arise in addition to the individual quasi-particle excitations when a quasi-particle accelerated in the medium [98, 102-106]. This leads to the conclusion given in [95, 101] that, in general, the Dirac quasi-particle is only an approximate description of an entire system with the collective excitations as the stable or unstable bound quasi-particle pairs. In a simple approximation there arise CP-odd excitations of zero mass as well as CP-even massive bound states of nucleon number zero and two. Along the same line we must substitute in eq.(12.6.5) the massless field  $\Psi^{(0)} \equiv \psi_Q \psi_\eta^{(0)} \psi_W$  by massive field  $\Psi^{(m)} \equiv \psi_Q \psi_\eta^{(m)} \psi_W$ . We obtain

$$\gamma p_Q \psi_Q^{(m)} = \Sigma_Q \psi_Q^{(m)}, \quad \gamma p_W \Psi^{(m)} = 0, \quad \gamma p_\eta \Psi^{(m)} = (\gamma p_Q + \gamma p_W) \Psi^{(m)} = \Sigma_Q \Psi^{(m)}. \quad (12.7.1)$$

Such redefinition  $\psi_Q^{(0)} \rightarrow \psi_Q^{(m)}$  leaves the structure of that piece of Lagrangian eq.(12.6.3) involving only the fields  $\psi_\eta$  and  $\psi_W$  unchanged

$$L_0 = L_{\eta_0}^{(0)} - L_{W_0}^{(0)} = \left( L_{\eta_0}^{(0)} - \Sigma_Q \bar{\Psi} \Psi \right) - \left( L_{W_0}^{(0)} - \Sigma_Q \bar{\Psi} \Psi \right) = L_{\eta_0}^{(m)} - L_{W_0}^{(m)}, \quad (12.7.2)$$

where the component  $\psi_Q$  is left implicit. Upon combining and rearranging relevant terms we separate the following pieces in the resulting gauge invariant Lagrangian eq.(12.6.3):

$$L(D)_\eta = \frac{i}{2} \{ \bar{\Psi}_\eta \gamma D_\eta \psi_\eta - \bar{\Psi}_\eta \gamma \overleftarrow{D}_\eta \psi_\eta \} - f_Q \bar{\Psi} \Psi - \frac{1}{2} Tr(\mathbf{B} \bar{\mathbf{B}}) \quad (12.7.3)$$

$$L(D)_W = \frac{i}{2} \{ \bar{\Psi}_W \gamma D_W \psi_W - \bar{\Psi}_W \gamma \overleftarrow{D}_W \psi_W \} - \Sigma_Q \bar{\Psi} \Psi - \frac{\lambda}{2} S_W S_W^+ - \frac{1}{2} Tr(\mathbf{B} \bar{\mathbf{B}}), \quad (12.7.4)$$

provided by  $f_Q \equiv \Sigma_Q(p_Q, m_Q, \lambda, \Lambda)$ ,  $\Psi = \psi_Q \psi_\eta \psi_W$ . The eq.(12.7.3) and eq.(12.7.4) are the Lagrangians that we shall be concerned within the following.

## 12.8 The electroweak symmetry; the P-violation and the Weinberg mixing angle

The microscopic approach creates a particular incentive for the pertinent concepts and ideas of the unified electroweak interactions. If, according to the subsec.12.3, one admit

at very beginning that the local rotations in the W-world are occurred only around two arbitrary axes, then one immediately concludes that under such circumstances the weak interacting particles could not be realized, because of the condition of MW connections eq.(12.2.5), which is not satisfied yet. That is, the Q- and W-worlds cannot be realized separately. Following the [2], a simple way of effecting a reconciliation is to assume that during a realization of weak interacting charged fermions, under the action of the Q-world, instead of initial symmetry the spanning of the  $W$ - world into the world of unified electroweak interaction  $W_{(3)}^{loc}$  took place, where the local rotations always occur around all three axes. Taking into account that at the very beginning all subquark fields in W-world are massless, we cannot rule out the possibility that they are transformed independently. On the other hand, when this situation prevails the spanning  $W_{(2)}^{loc} \rightarrow W_{(3)}^{loc}$  must be occurred compulsory in order to provide a necessary background for the condition eq.(12.2.5) to be satisfied. The most likely attitude here is that doing away this shortage the subquark fields  $q_{L_1}, q_{L_2}, q_{R_1}$ , and  $q_{R_2}$  tend to give rise to triplet. The three dimensional effective space  $W_{(3)}^{loc}$  will then arise

$$W_{(2)}^{loc} \ni q_{(2)}^w (\vec{T}^w = \frac{1}{2}) \rightarrow q_{(3)}^w = \begin{pmatrix} q_R(\vec{T}^w = 0) \\ q_L(\vec{T}^w = \frac{1}{2}) \end{pmatrix} = \begin{pmatrix} q_3^w \\ q_1^w \\ q_2^w \end{pmatrix} \equiv \begin{pmatrix} q_{R_2} \\ q_{L_1} \\ q_{L_2} \end{pmatrix} \in W_{(3)}^{loc}. \quad (12.8.1)$$

The latter holds if violating initial P-symmetry the components  $q_{R_1}, q_{R_2}$  still remain in isosinglet states, namely the components  $q_L$  form isodoublet while  $q_R$  is a isosinglet:  $q_L(\vec{T}^w = \frac{1}{2}), q_R(\vec{T}^w = 0)$ , i.e., the mirror symmetry is broken. Corresponding local transformations are implemented upon triplet  $q_{(3)}^{w'} = f_W^{(3)} q_{(3)}^w$ , where making use of incompatibility relations eq.(2.5.2) one gets the expanded group of local rotations in W-world (see [2]):

$$f_{exp}^{(3)} = \begin{pmatrix} e^{-i\beta} & 0 & 0 \\ 0 & f_{11}e^{-i\frac{\beta}{2}} & f_{12}e^{-i\frac{\beta}{2}} \\ 0 & f_{21}e^{-i\frac{\beta}{2}} & f_{22}e^{-i\frac{\beta}{2}} \end{pmatrix} \in SU^{loc}(2)_L \otimes U^{loc}(1), \quad (12.8.2)$$

where  $U = e^{-i\vec{T}^w \vec{\theta}^w} \in SU^{loc}(2)_L$ ,  $U_1 = e^{-iY^w \theta_1} \in U^{loc}(1)$ . Here  $U^{loc}(1)$  is the group of weak hypercharge  $Y^w$  taking the following values for left- and right-handed subquark fields:  $q_R : Y^w = 0, -2$ ,  $q_L : Y^w = -1$ . Whence  $q_{(3)}^{w'} = f_{exp}^{(3)} q_{(3)}^w$  or

$$q'_L = e^{-i\vec{T}^w \vec{\theta}^w - iY_L^w \theta_1} q_L, \quad q'_R = e^{-iY_R^w \theta_1} q_R.$$

The spanning eq.(12.8.1) implies the P-violation in the W-world expressed in the reduction of initial symmetry group of local transformations of right-handed components  $q_R$ :

$$[SU(2)]_R \rightarrow [U(1)]_R, \quad (12.8.3)$$

where subscript  $(R)$  specified the transformations implemented upon right-handed components. The invariance of physical system of the fields  $q_R$  under initial group  $[SU(2)]_R$  may be realized as well by introducing non-Abelian massless vector gauge fields  $\mathbf{A} = \vec{T}^w \vec{A}$  with the values in Lie algebra of the group  $[SU(2)]_R$ . Under a reduction eq.(12.8.3) the

coupling constant ( $g$ ) changed into ( $g'$ ) specifying the interaction strength between  $q_R$  and the Abelian gauge field  $B$  associated with the local group  $[U(1)]_R$ . Thereto  $g = g' \tan \theta_w$ , where  $\theta_w$  is the Weinberg mixing angle. To define it we consider the interaction vertices corresponding to the groups  $[SU(2)]_R : g \mathbf{A} \bar{q}_R \gamma \frac{\tau}{2} q_R$  and  $[U(1)]_R : g' B \bar{q}_R \gamma \frac{Y^w}{2} q_R$ . Note that  $\frac{\lambda_8}{2}$  is in the same normalization scale as each of the matrices  $\frac{\lambda_i}{2}$  ( $i = 1, 2, 3$ ) :  $Tr \left( \frac{\lambda_8}{2} \right)^2 = Tr \left( \frac{\lambda_i}{2} \right)^2 = \frac{1}{2}$ . Hence, the vertex scale reads  $(\text{Scale})_{SU(2)} = g \frac{\lambda_3}{2}$ , which is equivalent to  $g \frac{\lambda_8}{2}$ . It is obvious that per generator scale could not be changed under the reduction eq.(12.10.1), i.e.  $\frac{(\text{Scale})_{SU(2)}}{N_{SU(2)}} = \frac{(\text{Scale})_{U(1)}}{N_{U(1)}}$ , where  $N_{SU(2)}$  and  $N_{U(1)}$  are the numbers of generators respectively in the groups  $SU(2)$  and  $U(1)$ . Thus,  $(\text{Scale})_{U(1)} = \frac{1}{3}(\text{Scale})_{SU(2)}$ . Stated somewhat differently, the normalized vertex for the group  $[U(1)]_R$  reads  $\frac{1}{3} g B \bar{q}_R \gamma \frac{\lambda_8}{2} q_R$ . In comparing the coefficients can then be equated  $\frac{g'}{g} = \tan \theta_w = \frac{1}{\sqrt{3}}$ . We may draw a statement that during the realization of MW- structure the spanning eq.(12.8.1) compulsory occurred, which is the source of P-violation in W-world incorporated with the reduction eq.(12.8.3). The latter is characterized by the Weinberg mixing angle with the value fixed at  $30^\circ$ .

## 12.9 Emergence of composite isospinor-scalar mesons

The field  $q_{(2)}^w$  is the W-component of total field  $q_{(2)} = \frac{q}{\eta}{}_{(2)} \frac{q}{W}{}_{(2)}$ , where the field component  $q$  is left implicit. Instead of it, below we introduce the additional suffix ( $Q = 0, \pm$ ) specifying electric charge of the field. At the very outset there is an absolute symmetry between the components  $q_1 = \frac{q_1}{\eta} \frac{q_1}{W}$  and  $q_2 = \frac{q_2}{\eta} \frac{q_2}{W}$ . Hence, left-and right-handed components of fields may be written

$$q_{1L} = \frac{q_{1L}^{(0)}}{\eta} \frac{q_{1L}^{(-)}}{W}, \quad q_{2L} = \frac{q_{2L}^{(-)}}{\eta} \frac{q_{2L}^{(0)}}{W}, \quad q_{1R} = \frac{q_{1R}^{(0)}}{\eta} \frac{q_{1R}^{(-)}}{W}, \quad q_{2R} = \frac{q_{2R}^{(-)}}{\eta} \frac{q_{2R}^{(0)}}{W}. \quad (12.9.1)$$

On the example of one lepton generation  $e$  and  $\nu$ , without loss of generality, in [2] we have exploited the properties of these fields. A further implication of other fermion generations is straightforward. We shown that the term  $-f_Q \bar{\Psi} \Psi$  arisen in the Lagrangian eq.(12.6.3) accommodates the Yukawa couplings between the fermions and corresponding isospinor-scalar mesons in fairly conventional form

$$-f_Q \bar{\Psi} \Psi = -f_e (\bar{L} H e_R + \bar{e}_R H^+ L) - f_\nu (\bar{L} H_c \nu_R + \bar{\nu}_R H_c^+ L), \quad (12.9.2)$$

where the charge conjugated field  $H_c$  is defined  $(H_c)_i = H^{*k} \varepsilon_{ik}$ , and the isospinor-scalar meson field  $H$  reads

$$H \equiv \gamma^0 \frac{q_L^+}{W} \frac{q_R}{W}, \quad H^+ \equiv \gamma^0 \frac{q_R^+}{W} \frac{q_L}{W}.$$

Then, the two possible composite isospinor-scalar mesons are as follows:

$$H_u = \begin{pmatrix} h_u^{(+)} \\ h_u^{(0)} \end{pmatrix}, \quad H_d = \begin{pmatrix} h_d^{(0)} \\ h_d^{(-)} \end{pmatrix},$$

where

$$\begin{aligned} h_u^+ &\equiv \left( q_{W1L}^{(-)} \right)^+ q_{W2R}^{(0)}, & h_u^0 &\equiv \left( q_{W2L}^{(0)} \right)^+ q_{W2R}^{(0)}, \\ h_d^0 &\equiv \left( q_{W1L}^{(-)} \right)^+ q_{W1R}^{(-)}, & h_d^- &\equiv \left( q_{W2L}^{(0)} \right)^+ q_{W1R}^{(-)}, \end{aligned}$$

In accordance with eq.(12.2.5), the isospinor-scalar meson carries following weak hypercharge  $H : Y^w = 1$ . To compute the coupling constants  $f_e$  and  $f_\nu$  for the leptons one must retrieve their implicit field-components  $\psi_Q$ . Hence

$$f_i = \text{tr}(\rho_i^Q \Sigma_Q), \quad f_i^\nu = \text{tr}(\rho_i^{Q\nu} \Sigma_Q), \quad (12.9.3)$$

where the density operators  $\rho_i^Q$  and  $\rho_i^{Q\nu}$  for given  $i$  of the pure ensembles are used

$$\begin{aligned} \rho_i^Q &= (q_1 q_2 q_3)_i^{Q+} (q_1 q_2 q_3)_i^Q, & \rho_i^{Q\nu} &= (q_i \bar{q}_i)^{Q+} (q_i \bar{q}_i)^Q, \\ \text{tr}(\rho_i^Q)^2 &= \text{tr}(\rho_i^Q) = 1, & \text{tr}(\rho_i^{Q\nu})^2 &= \text{tr}(\rho_i^{Q\nu}) = 1. \end{aligned} \quad (12.9.4)$$

According to eq.(12.2.6), one gets

$$f_i = L_i^2 \bar{\Sigma}_Q, \quad f_i^\nu = L_i^2 \bar{\Sigma}_Q^\nu, \quad \bar{\Sigma}_Q \equiv \Sigma_Q(\lambda, L) \rho^Q, \quad \bar{\Sigma}_Q^\nu \equiv \Sigma_Q(\lambda, L) \rho^{Q\nu}, \quad (12.9.5)$$

where

$$\rho^Q = (q_1 q_2 q_3)^{Q+} (q_1 q_2 q_3)^Q, \quad \rho^{Q\nu} = (q \bar{q})^{Q+} (q \bar{q})^Q. \quad (12.9.6)$$

An implication of the quarks into this scheme is straightforward if one retrieves their implicit field-components  $\psi_Q, \psi_B, \psi_C$ , ( $C = s, c, b, t$ ) (see subsec.12.6). On the analogy of previous case the coupling constants read

$$f_i = \text{tr}(\rho_i \Sigma_Q), \quad (12.9.7)$$

where  $i = u, d, s, c, b, t$ . Taking into account the MW- structure of the quarks eq.(12.5.1), we may write down the corresponding density operators

$$\rho_i = \rho_i^Q \rho_i^B \rho_i^C \quad (12.9.8)$$

given in a convention

$$\rho_i^A = \psi_i^+ \psi_i, \quad (12.9.9)$$

where  $\rho_u^C = \rho_d^C = 1$ .

## 12.10 The Higgs boson

Within our approach the self-interacting isospinor-scalar Higgs bosons arise as the collective modes of excitations of bound quasi-particle iso-pairs. Pursuing the analogy with [96-109] in outlined here approach a key problem is to find out the eligible mechanism leading to the formation of pairs, somewhat like Cooper mechanism, but generalized for relativistic fermions, of course, in absence of any lattice. We suggested this mechanism in the framework of gauge invariance incorporated with the P-violation phenomenon in W-world [2]. To trace a maximum resemblance to the superconductivity theory, in this section it will be advantageous to describe our approach in terms of four dimensional

Minkowski space  $M_4$  corresponding to internal W-world:  $G \xrightarrow{W} M_4$  (subsec.2.1). Although we shall leave the suffix (W) implicit, but it goes without saying that all results obtained within this section refer to W-world. Following the previous section, we consider the isospinor-scalar  $H$ -meson arisen in W-world

$$H(x) = \gamma^0 \Psi_L^+(x) \Psi_R(x),$$

where  $x \in M_4$  is a point of W-world. The standard notational conventions will be employed throughout

$$q_L \equiv \psi_L(x) \rightarrow \Psi_L(x), \quad M_4 \rightarrow M_4, \quad q_R \equiv \psi_R(x) \rightarrow \Psi_R(x),$$

where  $\Psi_R(x) = \gamma(1 + \vec{\sigma}\vec{\beta})\Psi_L(x)$ ,  $\vec{\beta} = \frac{\vec{v}}{c}$ ,  $\Psi_L(x) = \gamma(1 - \vec{\sigma}\vec{\beta})\Psi_R(x)$ ,  $\gamma = \frac{E}{m}$ , provided by the spin  $\vec{\sigma}$ , energy  $E$  and velocity  $\vec{v}$  of particle. In terms of Fourier integrals

$$\Psi_L(x) = \frac{1}{(2\pi)^4} \int \Psi_L(p_L) e^{ip_L x} d^4 p_L, \quad \Psi_R(x) = \frac{1}{(2\pi)^4} \int \Psi_R(p_R) e^{ip_R x} d^4 p_R, \quad (12.10.1)$$

it is readily to get

$$H(k) = \int H(x) e^{-ikx} d^4 x = \gamma^0 \int \frac{d^4 p_L}{(2\pi)^4} \Psi_L^+(p_L) \Psi_R(p_L + k) = \gamma^0 \int \frac{d^4 p_R}{(2\pi)^4} \Psi_L^+(p_R - k) \Psi_R(p_R) \quad (12.10.2)$$

provided by conservation law of fourmomentum  $k = p_R - p_L$ , where  $k = k(\omega, \vec{k})$ ,  $p_{L,R} = p_{L,R}(E_{L,R}, \vec{p}_{L,R})$ . Our arguments on Bose-condensation are based on the local gauge invariance of the theory incorporated with the P-violation in weak interactions. The rationale for this approach is readily forthcoming from the consideration of gauge transformations of the fields eq.(12.10.1) under the P-violation in W-world

$$\Psi'_L(x) = U_L(x) \Psi_L(x), \quad \Psi'_R(x) = U_R(x) \Psi_R(x),$$

where the Fourier expansions have carried out over corresponding gauge quanta with wave fourvectors  $q_L$  and  $q_R$

$$U_L(x) = \int \frac{d^4 q_L}{(2\pi)^4} e^{iq_L x} U_L(q_L), \quad U_R(x) = \int \frac{d^4 q_R}{(2\pi)^4} e^{iq_R x} U_R(q_R), \quad (12.10.3)$$

and  $U_L(x) \neq U_R(x)$ . They induce the gauge transformations implemented upon  $H$ -field  $H'(x) = U(x) H(x)$ . The matrix of *induced gauge transformations* may be written down in terms of *induced gauge quanta*

$$U(x) \equiv U_L^+(x) U_R(x) = \int \frac{d^4 q}{(2\pi)^4} e^{iqx} U(q), \quad (12.10.4)$$

where  $q = -q_L + q_R$ ,  $q(q^0, \vec{q})$ . In momentum space one gets

$$H'(k') = \int \frac{d^4 q}{(2\pi)^4} U(q) H(k' - q) = \int \frac{d^4 k}{(2\pi)^4} U(k' - k) H(k). \quad (12.10.5)$$

Conservation of fourmomentum requires that  $k' = k + q$ . According to eq.(12.10.2) and eq.(12.10.5), we have

$$-p'_L + p'_R = -p_L + p_R + q = -p''_L + p_R = -p_L + p''_R,$$

where  $p''_L = p_L - q$ ,  $p''_R = p_R + q$ . As to the wave vectors of fermions, they imply the conservation law  $\vec{p}_L + \vec{p}_R = \vec{p}'_L + \vec{p}'_R$  characterizing the scattering process of two fermions with *effective interaction caused by the mediating induced gauge quanta*. We suggest the mechanism for the effective attraction between the fermions in the following manner: among all induced gauge transformations with miscellaneous gauge quanta we distinguish a subset with the induced gauge quanta of the frequencies characterized by the maximum frequency  $\frac{\tilde{q}}{\hbar}$  ( $\tilde{q} = \max\{q^0\}$ ) greater than the frequency of inducing oscillations fermion force  $\frac{\bar{E}_L - \bar{E}''_L}{\hbar} < \frac{\tilde{q}}{\hbar}$ . To the extent that this is a general phenomenon, we can expect under this condition the effective attraction (negative interaction) arisen between the fermions caused by exchange of virtual induced gauge quanta if only the forced oscillations of these quanta occur in the same phase with the oscillations of inducing force (the oscillations of fermion density). In view of this we may think of isospinor  $\Psi_L$  and isoscalar  $\Psi_R$  fields as the fermion fields composing the iso-pairs with the same conserving net momentum  $\vec{p} = \vec{p}_L + \vec{p}_R$  and opposite spin, for which the maximum number of negative matrix elements of operators composed by corresponding creation and annihilation operators  $a_{\vec{p}''_R}^+ a_{\vec{p}_R} a_{\vec{p}''_L}^+ a_{\vec{p}_L}$  (designated by the pair wave vector  $\vec{p}$ ) may be obtained for coherent ground state with  $\vec{p} = \vec{p}_L + \vec{p}_R = 0$ . The fermions filled up the Fermi sea block the levels below Fermi surface. Hence, the fermions are in superconductive or normal state described by Bloch individual particle model. Thus, the Bose-condensate arises in the W-world as the collective mode of excitations of bound quasi-particle iso-pairs described by the same wave function in the superconducting phase  $\Psi = \langle \Psi_L \Psi_R \rangle$ , where  $\langle \dots \rangle$  is taken to denote the vacuum averaging. The vacuum of the W-world filled up by such iso-pairs at absolute zero  $T = 0$ .

We make a final observation that  $\Psi_R \Psi_R^+ = n_R$  is a scalar density number of right-handed particles. Then it readily follows:

$$(\Psi_L \Psi_R)^+ (\Psi_L \Psi_R) = H H^+, \quad (12.10.6)$$

where  $|\Psi|^2 = \langle H H^+ \rangle = |\langle H \rangle|^2 \equiv |H|^2$ . It is convenient to abbreviate the  $\langle H \rangle$  by the symbol  $H$ . Thus, the  $H$ -meson actually arises as the collective mode of excitations of bound quasi-particle iso-pairs.

## 12.11 The energy gap function

We start with total Lagrangian eq.(12.7.4) of self-interacting fermion field in W-world

$$\begin{aligned} L_W(x) = & \frac{i}{2} \{ \bar{\Psi}_W(x) \gamma^\mu \partial_{W\mu} \psi_W(x) - \bar{\Psi}_W(x) \gamma^\mu \overleftarrow{\partial}_{W\mu} \psi_W(x) \} - m \bar{\Psi}_W(x) \psi_W(x) - \\ & - \frac{\lambda}{2} \bar{\Psi}_W(x) \left( \bar{\Psi}_W(x) \psi_W(x) \right) \psi_W(x), \end{aligned} \quad (12.11.1)$$

where,  $m = \Sigma_Q$  is the self-energy operator of the fermion field component in Q-world, the suffix ( $W$ ) just was put forth for instance in illustration of a point at issue. For the sake of



simplicity, we also admit  $\mathbf{B}(x) = 0$ , but of course one is free to restore the gauge field  $\mathbf{B}(x)$  whenever it should be needed. In lowest order the relation  $m \equiv m_Q \ll \lambda^{-1/2}$  holds. At non-relativistic limit the function  $\Psi$  reads  $\Psi \rightarrow e^{imc^2t}\Psi$ , and Lagrangian eq.(12.11.1) leads to Hamiltonian used in [99]. We make use of the Gor'kov's technique and evaluate the field equations ensued from the eq.(12.11.1) in following manner: The spirit of the calculation will be to treat interaction between the particles as being absent everywhere except the thin spherical shell  $2\tilde{q}$  near the Fermi surface. The Bose condensate of bound particle iso-pairs occurred at zero momentum. The scattering processes between the particles are absent. We consider the matrix elements  $\langle T(\Psi_\alpha(x_1)\Psi_\beta(x_2)\bar{\Psi}_\gamma(x_3)\bar{\Psi}_\delta(x_4)) \rangle$  and introduce the functions

$$\begin{aligned} \langle N | T(\gamma^0\Psi(x)\Psi(x')) | N+2 \rangle &= e^{-2i\mu't}F(x-x'), \\ \langle N+2 | T(\Psi^+(x)\gamma^0\Psi^+(x')) | N \rangle &= e^{2i\mu't}F^+(x-x'). \end{aligned} \quad (12.11.2)$$

Here,  $\mu' = \mu + m$ ,  $\mu$  is the chemical potential. We omit a prime over  $\mu$ , but should understand  $\mu + m$  under it. Making use of Fourier integrals, it renders the field equations easier to handle in momentum space [2]

$$\begin{aligned} (\gamma p - m)G(p) - i\lambda\gamma^0 F(0+)\bar{F}(p) &= 1, \\ \bar{F}(p)(\gamma p + m - 2\mu\gamma^0) + i\lambda\bar{F}(0+)G(p) &= 0, \end{aligned} \quad (12.11.3)$$

where  $G(p)$  is the thermodynamic Green's function,  $F_{\alpha\beta}(0+) = e^{2i\mu t} \langle T(\gamma^0\Psi_\alpha(x)\Psi_\beta(x)) \rangle = \lim_{x \rightarrow x'(t \rightarrow t')} F_{\alpha\beta}(x-x')$ . Next we substitute

$$(\gamma p - m) = (\omega' - \xi_p)\gamma^0, \quad (\gamma p + m - 2\mu\gamma^0) = \gamma^0(\omega' + \xi_p^+),$$

where

$$\begin{aligned} \omega' &= \omega - \mu' = \omega - m - \mu, \quad \xi_p = (\vec{\gamma}\vec{p} + m)\gamma^0 - \mu' = (\vec{\gamma}\vec{p} + m)\gamma^0 - m - \mu, \\ \xi_p^+ &= \gamma^0(\vec{\gamma}^+\vec{p} + m) - m - \mu, \end{aligned}$$

and omit a prime over  $\omega'$  for the rest of this section. We apply

$$F(0+) = -JI, \quad I = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad F^+(0+)F(0+) = -J^2I^2 = J^2,$$

and  $\hat{\omega} + \hat{\xi}_p = \gamma^0(\omega + \xi_p)$ . The gap function  $\Delta$  reads  $\Delta^2 = \lambda^2 J^2$ , where  $J = \int \frac{d\omega d\vec{k}}{(2\pi)^4} F^+(p)$ .

Using the standard rules [108], one may pass over the poles. This method allows oneself to extend the study up to limit of temperatures, such that  $T_c - T \ll T_c$  in terms of thermodynamic Green's function. So

$$\begin{aligned} F^+(p) &= -i\lambda J(\omega - \xi_p + i\delta)^{-1}(\omega + \xi_p - i\delta)^{-1} - \frac{\pi\Delta}{\varepsilon_p} n(\varepsilon_p) \{ \delta(\omega - \varepsilon_p) + \delta(\omega + \varepsilon_p) \}, \\ G(p) &= \gamma^0 \{ u_p^2(\omega - \xi_p + i\delta)^{-1} + v_p^2(\omega + \xi_p - i\delta)^{-1} + 2\pi i n(\varepsilon_p) [u_p^2\delta(\omega - \varepsilon_p) - v_p^2\delta(\omega + \varepsilon_p)] \}, \end{aligned} \quad (12.11.4)$$

where  $u_p^2 = \frac{1}{2} \left( 1 + \frac{\xi_p}{\varepsilon_p} \right)$ ,  $v_p^2 = \frac{1}{2} \left( 1 - \frac{\xi_p}{\varepsilon_p} \right)$ ,  $\varepsilon_p = (\xi_p^2 + \Delta^2(T))^{1/2}$ . and  $n(\varepsilon_p)$  is the usual Fermi function  $n(\varepsilon_p) = \left( \exp \frac{\varepsilon_p}{T} + 1 \right)^{-1}$ . Then

$$1 = \frac{|\lambda|}{2(2\pi)^3} \int d\vec{k} \frac{1 - 2n(\varepsilon_k)}{\varepsilon_k(T)} \quad (|\xi_p| < \tilde{q}), \quad (12.11.5)$$

determining the energy gap  $\Delta$  as a function of  $T$ . According to eq.(12.11.5), the  $\Delta(T) \rightarrow 0$  at  $T \rightarrow T_c \sim \Delta(0)$  [96].

## 12.12 Self-interacting potential of Bose-condensate

To go any further in exploring the form and significance of obtained results it is entirely feasible to include the generalization of the equations (12.11.3) in presence of spatially varying magnetic field with vector potential  $\vec{A}(\vec{r})$ , which is straightforward ( $t \rightarrow \tau = it$ )

$$\begin{aligned} & \left\{ -\gamma^0 \frac{\partial}{\partial \tau} - i\vec{\gamma} \left( \frac{\partial}{\partial \vec{r}} - ie\vec{A}(\vec{r}) \right) - m + \gamma^0 \mu \right\} G(x, x') + \gamma^0 \Delta(\vec{r}) \bar{F}(x, x') = \delta(x - x'), \\ & \bar{F}(x, x') \left\{ \gamma^0 \frac{\partial}{\partial \tau} + i\vec{\gamma} \left( \frac{\partial}{\partial \vec{r}} + ie\vec{A}(\vec{r}) \right) - m + \gamma^0 \mu \right\} - \Delta^*(\vec{r}) \gamma^0 G(x, x') = 0, \end{aligned} \quad (12.12.1)$$

where the thermodynamic Green's function [116,117] is used, the energy gap function is in the form  $\Delta^*(\vec{r}) = \lambda F^+(\tau, \vec{r}; \tau, \vec{r})$ . This function is logarithmically divergent, but with a cutoff of energy of interacting fermions at the spatial distances in order of  $\frac{\hbar v}{\tilde{\omega}}$  can be made finite, where  $\tilde{\omega} \equiv \frac{\tilde{q}}{\hbar}$ . To handle the eq.(12.12.1) one uses the Fourier components of functions  $G(x, x')$  and  $F(x, x')$

$$G(\vec{r}, \vec{r}'; u) = T \sum_n e^{-i\omega u} G_\omega(\vec{r}, \vec{r}'), \quad G_\omega(\vec{r}, \vec{r}') = \frac{1}{2} \int_{-1/T}^{1/T} e^{i\omega u} G(\vec{r}, \vec{r}'; u) du, \quad (12.12.2)$$

where  $u = \tau - \tau'$ ,  $\omega$  is the discrete index  $\omega = \pi T(2n + 1)$ ,  $n = 0, \pm 1, \dots$  the gap function is defined by  $\Delta^*(\vec{r}) = \lambda T \sum_n F_\omega^+(\vec{r}, \vec{r}')$ . Then (see [2])

$$G_\omega(\vec{r}, \vec{r}') = \tilde{G}_\omega(\vec{r}, \vec{r}') - \int \tilde{G}_\omega(\vec{r}, \vec{s}) \gamma^0 \Delta(\vec{s}) \bar{F}_\omega(\vec{s}, \vec{r}') d^3 s, \quad (12.12.3)$$

and

$$\bar{F}_\omega(\vec{r}, \vec{r}') = \int \tilde{G}_\omega(\vec{s}, \vec{r}') \Delta^*(\vec{s}) \gamma^0 \tilde{G}_{-\omega}(\vec{s}, \vec{r}) d^3 s. \quad (12.12.4)$$

where  $\tilde{G}_\omega(\vec{r}, \vec{r}')$  is the Bloch individual particle Green's function of the fermion in normal mode. The gap function  $\Delta(\vec{r})$  as well as  $\bar{F}_\omega(\vec{r}, \vec{r}')$  are small ones at close neighbourhood of transition temperature  $T_c$  and varied slowly over a coherence distance. This approximation, which went into the derivation of equations, meets our interest in eq.(12.12.3), eq.(12.12.4). Using standard procedure one may readily express them in power series of  $\Delta$  and  $\Delta^*$  by keeping only the terms in  $\bar{F}_\omega(\vec{r}, \vec{r}')$  up to the cubic and in  $G_\omega(\vec{r}, \vec{r}')$  - quadratic order in  $\Delta$ . The technique now is to average over the polarization of particles and expand

the obtained equation up to the terms quadratic in  $(\vec{r} - \vec{r}')$ . Keeping in mind aforesaid, after calculations the resulting equation can be written [2]

$$\left\{ \left( i\hbar \vec{\nabla} + \frac{e^*}{c} \vec{A} \right)^2 + \frac{2m}{\nu} \left[ \frac{2\pi^2}{\lambda m p_0} \left( \frac{\mu}{m} \right)^2 \left( \frac{\mu}{m} - 1 \right) + \left( \frac{\mu}{m} \right)^2 \left( \frac{T}{T_{c\mu}} - 1 \right) + \frac{2}{N} |\Psi(\vec{r})|^2 \right] \right\} \Psi(\vec{r}) = 0, \quad (12.12.5)$$

where  $\nu = \frac{7\zeta(3)mv_F^2}{24(\pi k_B T_c)^2}$  and  $T_{c\mu} = \frac{m}{\mu} T_c$ . Succinctly

$$\left\{ \vec{p}_A^2 - \frac{1}{2} m_\Psi^2 + \frac{1}{4} \lambda_\Psi^2 |\Psi(\vec{r})|^2 \right\} \Psi(\vec{r}) = 0, \quad (12.12.6)$$

provided by

$$\begin{aligned} m_\Psi^2(\lambda, T, T_{c\mu}) &= \frac{24}{7\zeta(3)} \left( \frac{\hbar}{\xi_0} \right)^2 \left( \frac{\mu}{m} \right)^2 \left[ 1 - \frac{T}{T_{c\mu}} - \left( \frac{\mu}{m} - 1 \right) \ln \frac{2\tilde{\omega}}{\Delta_0} \right], \\ \lambda_\Psi^2(\lambda, T_c) &= \frac{96}{7\zeta(3)} \left( \frac{\hbar}{\xi_0} \right)^2 \frac{1}{N}, \quad \Psi(\vec{r}) = \Delta(\vec{r}) \frac{(7\zeta(3)N)^{1/2}}{4\pi k_B T_c}, \quad \vec{p}_A = i\hbar \vec{\nabla} + \frac{e^*}{c} \vec{A}. \end{aligned} \quad (12.12.7)$$

Whence, the transition temperature decreases inversely by relativistic factor  $\frac{\mu}{m}$ . A spontaneous breaking of symmetry of ground state occurs at  $\eta_\Psi^2(\lambda, T < T_{c\mu}) > 0$ , where  $\eta_\Psi^2(\lambda, T, T_{c\mu}) = \frac{m_\Psi^2}{\lambda_\Psi^2}$ .

The eq.(12.12.6) splits into the couple of equations for  $\Psi_L$  and  $\Psi_R$ . Thus, a Lagrangian of the  $H$  boson will be arisen with the corresponding values of mass  $m_\Psi^2 \equiv m_H^2$  and coupling constant  $\lambda_\Psi^2 \equiv \lambda_H^2$ .

### 12.13 Four-component Bose-condensate in magnetic field

In [2] we have derived the equation of four-component bispinor field of Bose-condensate, wherein due to self-interaction the spin part of it is vanished. Actually, we start with the nonsymmetric state  $\Delta_L \neq \Delta_R$ , where  $\Psi_L$  and  $\Psi_R$  are two eigenstates of chirality operator  $\gamma_5$ . Then, the eq.(12.12.5) enables to postulate the equation of four-component Bose-condensate in equilibrium state at presence of the magnetic field

$$i\hbar \frac{\partial \Psi}{\partial t} = \left\{ c\vec{\alpha} \left( \vec{p} + \frac{e^*}{c} \vec{A} \right) + \beta m c^2 + M(F) + L(F) |\Psi|^2 \right\} \Psi = 0, \quad (12.13.1)$$

where the functions  $M(F)$  and  $L(F)$  can be determined under the requirement that the second-order equations ensued from the eq.(12.13.1) must match onto eq.(12.12.6). Also, taking into account an approximation fitting our interest that the gap function is small at close neighbourhood of transition temperature, we get:

$$\begin{aligned} M(F) &= \left( M_0^2 + \frac{i}{2} e^* F \right)^{1/2}, \quad M_0 = \left( m^2 + \frac{1}{2} m_H^2 \right)^{1/2}, \quad L(F) = -\frac{\lambda_H^2}{8M(F)}, \\ L_0 &= -\frac{\lambda_H^2}{8M_0}, \quad M(F)L(F) = M_0 L_0 = -\frac{1}{8} \lambda_H^2, \end{aligned}$$

such that

$$\left\{ \bar{p}_A^2 + m^2 - \left( M_0 + L_0 |\Psi|^2 \right)^2 \right\} \Psi(\vec{r}) \equiv \left\{ \bar{p}_A^2 - \frac{1}{2} m_H^2 + \frac{1}{4} \lambda_H^2 |\Psi|^2 \right\} \Psi(\vec{r}) = 0. \quad (12.13.2)$$

This has yet another important consequence. At  $\Delta_L \neq 0$  and imposed constraint  $(m + M(F) + L(F) |\Psi|^2)_{F \rightarrow 0} \rightarrow 0$  we have

$$\Delta_2 = \frac{1}{\sqrt{2}} (\Delta_L - \Delta_R) = 0, \quad \Psi_2 = 0. \quad (12.13.3)$$

Thus, the  $|\Psi_0|$  is the gap function symmetry-restoring value

$$\Delta_2 \left( |\Psi_0|^2 = \frac{m + M_0}{-L_0} \right) = 0, \quad \Delta_L \left( |\Psi_0|^2 \right) = \Delta_R \left( |\Psi_0|^2 \right),$$

where, according to eq.(12.13.2), one has

$$V \equiv \left[ m^2 - \left( M_0 + L_0 |\Psi|^2 \right)^2 \right] \Psi^2 = \left[ -\frac{1}{2} m_H^2 + \frac{1}{4} \lambda_H^2 |\Psi|^2 \right] \Psi^2, \quad (12.13.4)$$

and

$$V \left( |\Psi_0|^2 = \frac{m + M_0}{-L_0} \right) = \frac{1}{2} \eta_H^2(\lambda, T, T_{c\mu}) = 0. \quad (12.13.5)$$

We conclude that the field of symmetry-breaking Higgs boson must be counted off from the  $\Delta_L = \Delta_R$  symmetry-restoring value of Bose-condensate  $|\Psi_0| = \frac{1}{\sqrt{2}} \eta_H(\lambda, T, T_{c\mu})$  as the point of origin describing the excitation in the neighbourhood of stable vacuum eq.(12.13.5). The gauge invariant Lagrangian eq.(12.7.4) takes the form

$$L_W(D) = \frac{i}{2} \left\{ \bar{\Psi} \gamma \frac{D}{W} \psi - \bar{\Psi} \gamma \overleftarrow{\frac{D}{W}} \psi \right\} - \bar{\Psi} \left\{ m + \gamma^0 [M(F) + L(F) |\Psi|^2] \right\} \frac{\psi}{W}. \quad (12.13.6)$$

At the symmetry-restoring point, this Lagrangian can be replaced by

$$L_W(D) \rightarrow L_{W_1}(D) = \frac{1}{2} \left( \frac{D}{W} \psi \right)^2 - V_W \left( \left| \frac{\psi}{W_1} \right|^2 \right),$$

provided

$$V_W \left( \left| \frac{\psi}{W_1} \right|^2 \right) = -\frac{1}{2} m_H^2 \psi^2 + \frac{1}{4} \lambda_H^2 \psi^4.$$

Taking into account the eq.(12.10.6), in which  $\left| \frac{\psi}{W_1} \right|^2 = \left| H \right|^2 = \frac{1}{2} |\eta_H + \chi|^2$ , one gets

$$L_{W_H}(D) = \frac{1}{2} \left( \frac{D}{W} H \right)^2 - V_W \left( |H|^2 \right), \quad V_W \left( |H|^2 \right) = -\frac{1}{2} m_H^2 \varphi^2 + \frac{1}{4} \lambda_H^2 H^4. \quad (12.13.7)$$

Finally, recording the question of whether or not it is possible to extend the ideas of former approach to lower temperatures as it was investigated in the case of Gor'kov's theory by others [112-114], in [2], as usual, we admit that the order parameter and vector potential vary slowly over distances of the order of the coherence length. We restrict ourselves to the London limit and the derivation of equations was proceeded by iterating to a low

order giving only the leading terms. Taking into account the eq.(12.12.1), eq.(12.12.3) and eq.(12.12.4), it is straightforward to derive the separate integral equations for  $G$  and  $F^+$  in terms of  $\Delta$ ,  $\Delta^*$  and  $\tilde{G}$ . The mathematical structure of obtained equations is closely similar to that studied by [112,119,120] in somewhat different context. Then, adopting their technique in [2] we introduce sum and difference coordinates, and Fourier transform with respect to the difference coordinates. To obtain resulting expressions we have proceeded with further standard calculations. There is only one thing to be noticed about the integration. Due to the angular integration in momentum space, the terms linear in the vector  $\vec{p}$  will be vanished, as well as the integration over the energies removes the linear terms in  $\epsilon(\vec{p}, \vec{R}) \equiv \frac{1}{2m} \left( \vec{p} - \frac{e}{c} \vec{A}(\vec{R}) \right)^2 - \mu_0$ . Thus, we may expand the quantities according to the degree of inhomogeneity somewhat like it we have done in equations (12.12.3) and eq.(12.12.4) of gap function  $\Delta^*(\vec{r})$ , which in mixed representation transforms to the following:

$$\Delta^*(\vec{R}) = T \sum_{\omega} \int \frac{d^3 p}{(2\pi)^3} F_{\omega}^+(\vec{p}, \vec{R}) = T \sum_{\omega} \int \frac{d^3 p}{(2\pi)^3} \hat{\gamma}_A(\vec{p}, \vec{R}) F_{\Omega}^+(\vec{p}, \vec{R}). \quad (12.13.8)$$

The approximation was used to obtain the function  $F_{\Omega}^+$  must be of one order higher  $F_{\Omega}^+ \simeq F_{\Omega}^{(0)+} + F_{\Omega}^{(1)+} + F_{\Omega}^{(2)+}$  than that for function  $\tilde{G}_{\Omega} \simeq \tilde{G}_{\Omega}^{(0)} + \tilde{G}_{\Omega}^{(1)}$ . Applying an iteration method of solution one replaces  $K \rightarrow \tilde{K}$ ,  $G \rightarrow \tilde{G}$  in eq.(12.17.5) and puts  $\Theta^{(0)} = 1$ ,  $\tilde{K}^{(1)} = 0$ ,  $\tilde{G}^{(1)} = 0$ . Hence  $\tilde{G} = \tilde{G}^{(0)}$ . The resulting equation for gap function is similar to those occurring in [112], although not identical. The sole difference is that in the resulting equation we use the expressions of  $\Omega$  and  $\xi$ .

## 12.14 Transmission of the electroweak symmetry breaking from the $W$ -world to spacetime continuum; the two solid phenomenological implications of the MSM

A common feature of gauge theories is that to break the gauge symmetry down and leading to masses of the fields, one needs in general, several kinds of spinless Higgs mesons, with conventional Yukawa couplings to fermion currents and transforming by an irreducible representation of gauge group. The conventional Higgs theory like [121] involves these bosons as the ready made fundamental elementary fields defined on the Minkowski spacetime continuum  $M_4$ , which entails various difficulties. As it is seen in the previous subsections, the self-interacting isospinor scalar Higgs bosons arise in MSM as the collective modes of excitations of bound quasi-particle iso-pairs in the internal  $W$ -world. Whence, a first important phenomenological implication of the MSM ensues at once that

- *such Higgs bosons never could emerge in spacetime continuum and, thus, could not be discovered in experiments nor at any energy range.*

It just remains to see how can these bosons break the gauge symmetry down in  $M_4$  and lead to masses of the spacetime-components of the MW-fields? It is remarkable to see that the MSM, in contrast to the SM, predicts the transmission of electroweak symmetry breaking from the  $W$ -world to the  $M_4$  spacetime continuum. Actually, in standard scenario for the simplest Higgs sector, a gauge invariance of the Lagrangian is broken in the  $W$ -world when the  $H$ -meson fields eq.(12.13.7) acquire a VEV  $\eta_H \neq 0$ . Thereby the mass  $m_H$  and coupling constant  $\lambda_H$  are in the form eq.(12.15.15). The spontaneous breakdown of

symmetry is vanished at  $\eta_H^2(\lambda, T > T_{c\mu}) < 0$ . When this doublet obtains a VEV, three of the gauge fields  $Z_W^0, W_W^\pm$  acquire masses. These fields are the  $W$ -components of the mesons mediating the weak interactions. Certainly, the derivative

$$D_\mu H \equiv \left( \partial_\mu - \frac{i}{2} g \tau \cdot \mathbf{W}_\mu - \frac{i}{2} g' X_\mu \right) H$$

arisen in the eq.(12.16.9), in standard scenario leads to the masses

$$M_W = \frac{g \eta_H}{2}, \quad M_Z^2 = \frac{(g^2 + g'^2)^{1/2} \eta_H}{2}, \quad \cos \theta_W = \frac{M_W}{M_Z},$$

respectively of the gauge field components

$$W_W^\pm = \frac{1}{\sqrt{2}} \left( W_W^1 \pm W_W^2 \right), \quad Z_W^\mu = \frac{g W_W^3 - g' X_\mu}{(g^2 + g'^2)^{1/2}} \equiv \cos \theta_W W_W^3 - \sin \theta_W X_\mu.$$

Consequently, a remaining massless gauge field

$$A_\mu = \frac{g' W_W^3 + g X_\mu}{(g^2 + g'^2)^{1/2}} \equiv \sin \theta_W W_W^3 + \cos \theta_W X_\mu.$$

will be identified as the  $W$ -component of the photon field coupled to the electric current. Therewith the  $x$ -components of the fields above simultaneously acquire corresponding masses too, since, according to the specific MW scheme (see subsec.12.7), all the components of corresponding frame fields are defined on the MW mass shells, i.e.,

$$\square_x W_\mu = M_W^2 W_\mu, \quad \square_x Z_\mu = M_Z^2 Z_\mu, \quad \square_x A_\mu = M_A^2 A_\mu,$$

provided by

$$M_W^2 W_\mu \equiv \square_W W_\mu, \quad M_Z^2 Z_\mu \equiv \square_W Z_\mu, \quad M_A^2 A_\mu \equiv \square_W A_\mu = 0.$$

The microscopic structure of these fields reads

$$W^+ \equiv \phi_W(\eta) (q_1 q_2 q_3)^Q (q\bar{q})^W, \quad W^- \equiv \phi_W(\eta) (\overline{q_1 q_2 q_3})^Q (\bar{q}q)^W, \\ Z^0 \equiv \phi_Z(\eta) (q\bar{q})^Q (q\bar{q})^W, \quad A \equiv \phi_A(\eta) (q\bar{q})^Q A_W(0).$$

The values of the masses  $M_W$  and  $M_Z$  are changed if the Higgs sector is built up more compoundly. Due to Yukawa couplings eq.(12.9.2) the fermions acquire the masses after symmetry-breaking. The mass of electron reads  $m_e = \frac{\eta_H}{\sqrt{2}} f_e$  etc. One gets for the leptons  $f_e : f_\mu : f_\tau = m_e : m_\mu : m_\tau$ . This mechanism does not disturb the renormalizability of the theory [122,123]. In approximation to lowest order  $f = \Sigma_Q \simeq m_Q \ll \lambda^{-1/2} \left( \lambda^{-1} = \frac{mp_0}{2\pi^2} \ln \frac{2\tilde{\omega}}{\Delta_0} \right)$ , the Lagrangians eq.(12.7.3) and eq.(12.7.4) produce the Lagrangian of phenomenological SM, where at  $f \sim 10^{-6}$  one gets  $\lambda \ll 10^{12}$ . In standard scenario the lowest pole  $m_Q$  of the self-energy operator  $\Sigma_Q$  has fixed the whole mass spectrum of the SM particles. But, in general, one must also take into account the mass

spectrum of expected various collective excitations of bound quasi-particle pairs produced by higher-order interactions as a “superconductive” solution obtained from a nonlinear spinor field Lagrangian of the  $Q$ -component possessed  $\gamma_5$  invariance. These states must be considered as a direct effect of the same primary nonlinear fermion interaction which provides the mass of the  $Q$ -component of Fermi field, which itself is a collective effect. They would manifest themselves as stable or unstable states. The general features of mass spectrum of the collective excitations and their coupling with the fermions are discussed in [95] through the use of the Bethe-Salpeter equation handled in the simplest ladder approximation incorporated with the self-consistency conditions, when one is still left with unresolved divergence problem. One can reasonably expect that these results for the bosons of small masses at low energy compared to the unbound fermion states are essentially correct in spite of the very simple approximations. Therein, some bound states are predicted too the obtained mass values of which are rather high, and these states should decay very quickly. The high-energy poles may in turn determine the low-energy resonances. All this prompt us to expect that the other poles different from those of lowest one in turn will produce the new heavy SM family partners. Hence one would expect a second important phenomenological implication of the MSM that:

- *for each of the three SM families of quarks and leptons there are corresponding heavy family partners with the same  $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$  quantum numbers at the energy scales related to next poles with respect to lowest one.*

To see its nature, now we may estimate the energy threshold of creation of such heavy family partners using the results far obtained in [95, 101]. It is therefore necessary under the simplifying assumption to consider in the  $Q$ -world a composite system of dressed fermion ( $N_*$ ) made of the unbound fermion ( $N$ ) coupled with the different kind two-fermion bound states ( $N\bar{N}$ ) at low energy, which all together represent the primary manifestation of the fundamental interaction. Such a dressed fermion would have a total mass  $m_* \simeq m_Q + \mu$ , where  $m_Q$  and  $\mu$  are the masses, respectively, of the unbound fermion and the bound state. According to the general discussion of the mass spectrum of the collective excitations given in [95], here we are interested only in the following low-energy bound states written explicitly in spectroscopic notation  $(^1S_0)_{N=0}$ ,  $(^1S_0)_{N=\pm 2}$ ,  $(^3P_1)_{N=0}$  and  $(^3P_0)_{N=0}$  with the expected masses  $\mu = 0$ ,  $> \sqrt{2}m_Q$ ,  $\sqrt{\frac{8}{3}}m_Q$  and  $2m_Q$ , respectively, where the subscript  $N$  indicates the nucleon number. One notes the peculiar symmetry existing between the pseudoscalar and the scalar states that the first has zero mass and binding energy  $2m_Q$ , while the opposite holds for the scalar state. When the next pole  $m_*$  to the lowest one  $m_Q$  will be switched on, then due to the Yukawa couplings the all fermions will acquire the new masses with their common shift  $\frac{m_*}{m_Q} \equiv 1 + k$  held upwards along the energy scale. To fix the energy threshold value all we have to do then is choose the heaviest member among the SM fermions, which is the top quark, and to set up the quite obvious formula

$$E \geq E_0 \equiv m_t c^2 = (1 + k) m_t c^2,$$

where  $m_t$  is the mass of the top quark. The top quark observed firstly in the two FNAL  $p\bar{p}$  collider experiments in 1995, has the mass turned out to be startlingly large  $m_t = (173.8 \pm 5.0) GeV/c^2$  compared to all the other SM fermion masses [125]. Thus, we get the most important energy threshold scale estimate where the heavy partners of the SM extra families of quarks and leptons likely would reside at:  $E_1 > (419.6 \pm 12.0) GeV$ ,  $E_2 =$

$(457.6 \pm 13.2) GeV$  and  $E_3 = (521.4 \pm 15.0) GeV$ , corresponded to the next nontrivial poles are written:  $k_1 > \sqrt{2}$ ,  $k_2 = \sqrt{8/3}$  and  $k_3 = 2$ , respectively. We recognize well that the general results obtained in [95], however, model-dependent and may be considerably altered, especially in the high energy range by using better approximation. In present state of the theory it seemed to be a bit premature to get exact high energy results, which will be important subject for the future investigations. But, in the same time we believe that the approximation used in [95] has enough accuracy for the low-energy estimate made above. Anyhow, it is for one thing, the new scale where the family partners reside will be much higher than the electroweak scale and thus these heavy partners lie far above the electroweak scale.

### 13 Emergence of two composite isospinor Higgs chiral supermultiplets

It is remarkable that just the two doublets of isospinor-scalar Higgs bosons arise (subsec.12.9), which are necessary in any self-consistent SUSY theory. One cannot write a SUSY version of Yukawa interactions of SM without at least a second Higgs doublet since there are three well known reasons for it [24-48].

- A model with a single Higgs doublet superfield suffers from quadratic divergences because the trace of the hypercharge generator does not vanish.

- Such a model has nonvanishing gauge anomalies associated with fermion triangle diagrams. The condition for cancellation of gauge anomalies include  $Tr[Y^3] = Tr[T_3^2 Y] = 0$ , where  $T_3$  and  $Y$  are the third component of weak isospin and the weak hypercharge, respectively, in a normalization where the electric charge is  $Q = T_3 + Y$ . The trace run over all of the left-handed Weyl fermionic degrees of freedom in the theory. In the SM, these conditions are already satisfied by the known quarks and leptons, thus SM is anomaly-free. In SUSY version the definite chirality of the supersymmetric partner of Higgs boson carries  $U(1)$  hypercharge  $Y = \frac{1}{2}$  or  $Y = -\frac{1}{2}$ . In either case alone, such a fermion upsets the anomaly cancellation condition by making a nonzero contribution to the trace. This can be avoided in the case of two Higgs supermultiplets, one with each of  $Y = \pm\frac{1}{2}$ .

- The masses of chiral fermions must be supersymmetric in conventional SUSY theory, i.e., they must originate from terms in the superpotential. In the SM the Higgs doublet (the complex conjugate of the doublet) can couple to the  $T_3 = +\frac{1}{2}$  ( $T_3 = -\frac{1}{2}$ ) fermions in a gauge invariant way. In SUSY version, however, Yukawa interactions come from a superpotential, which cannot depend on a field as well as its complex conjugate. hence, any doublet can give mass either to a  $T_3 = +\frac{1}{2}$  or  $T_3 = -\frac{1}{2}$  fermion, but not both. Thus, to give masses to all the fermions one must introduce a second doublet  $H_d$ .

According to subsec.12.9 the Higgs bosons are composites

$$H = \gamma^0 \frac{\psi}{W}^\dagger_{DL} \frac{\psi}{W}_{DR} = \gamma^0 \frac{\bar{\chi}}{W} \frac{\bar{\psi}}{W}, \quad (13.1)$$

where the left  $\frac{\bar{\chi}}{W}$  and right-handed  $\frac{\bar{\psi}}{W}$  Weyl spinors are the members of chiral and anti-chiral superfields, respectively. Since they undergone SUSY transformations then Higgs



bosons also have their SUSY partners. The  $H$  belongs to chiral superfield, where its SUSY partner is Higgsino. Using eq.(13.1) and SUSY transformations of the components  $\bar{\chi}_W$  and  $\bar{\psi}_W$  one can obtain the explicit expression of the Higgsino. But it is much easier to handle it up, if one uses the SUSY transformation for the total Higgs boson itself as a member of chiral multiplet

$$\delta_\xi H = \sqrt{2} \xi \widetilde{H}, \quad (13.2)$$

where  $\widetilde{H}$  is the Higgsino. By means of eq.(13.1) and eq.(11.33) one gets

$$H + H^+ = \gamma^0 \left( \bar{\chi}_W \bar{\psi}_W + \chi_W \psi_W \right) = \gamma^0 i \left( \frac{2}{\lambda_W} \right)^{1/2} F_W, \quad (13.3)$$

namely

$$\delta_\xi (H + H^+) = \gamma^0 i \left( \frac{2}{\lambda_W} \right)^{1/2} \delta_\xi F_W, \quad (13.4)$$

or

$$\xi \widetilde{H} + (\xi \widetilde{H})^+ = i \gamma^0 \left( \frac{2}{\lambda_W} \right)^{1/2} \left( -i \bar{\xi} \bar{\sigma}^m \partial_m \bar{\chi}_W \right) \quad (13.5)$$

The Fiertz identity  $\bar{\xi} \bar{\sigma}^m \partial_m \bar{\chi}_W = - \partial_m \bar{\chi}_W \sigma^m \bar{\xi}$  gives

$$\widetilde{H}^+ = -\gamma_0 \partial_m \bar{\chi}_W \sigma^m = - \left( \bar{\sigma}^m \partial_m \chi_W \gamma_0 \right)^+.$$

Thus, explicitly the spinor- $\frac{1}{2}$  Higgsino doublets read

$$\widetilde{H}_u = -\bar{\sigma}^m \left( \partial_m \chi_W^u \right) \gamma_0, \quad \widetilde{H}_d = -\bar{\sigma}^m \left( \partial_m \chi_W^d \right) \gamma_0. \quad (13.6)$$

## 14 The superfield content of MSMSM and the resulting SUSY Lagrangian

The results obtained in the previous sections enable us to trace unambiguously rather general scheme of MSMSM, which is essentially a straightforward and viable supersymmetrization of the MSM where we want to keep the number of superfields and interactions as small as possible. To build up the MSMSM the major point is to define its superfield content. Below we recall some important features allowing us to write the resulting Lagrangian of MSMSM based on eq.(11.35).

- Within the MSM, during the realization of MW connections of weak interacting fermions the P-violation compulsory occurred in W-world (subsec.12.8) incorporated with the symmetry reduction eq.(12.8.3). It has characterized by the Weinberg mixing angle with the fixed value at  $30^\circ$ . This gives rise to the local symmetry  $SU(2) \otimes U(1)$ , under which the left-handed fermions transformed as six independent doublets, while the right-handed fermions transformed as twelve independent singlets.

- Due to vacuum rearrangement in Q-world the Yukawa couplings arise between the fermion fields and corresponding isospinor-scalar  $H$ -mesons in conventional form.

- In the framework of suggested mechanism, providing the effective attraction between the relativistic fermions caused by the exchange of the mediating induced gauge quanta

in W-world, the two complex self-interacting isospinor-scalar Higgs doublets ( $H_u, H_d$ ) as well as their spin- $\frac{1}{2}$  SUSY partners ( $\widetilde{H}_u, \widetilde{H}_d$ ) Higgsinos arise as the Bose-condensate. Taking into account this slight difference from the MSM arisen in the field content of MSMSM in the Higgs sector, we must explicitly write in the supersymmetric Lagrangian eq.(11.35) also the piece containing these fields coupled to the gauge fields in a gauge invariant way, when the symmetry-breaking Higgs bosons are counted off from the gap symmetry-restoring value as the point of origin (subsec.12.13).

- The gauge group of MSMSM is the same  $SU_c(3) \otimes SU(2)_L \otimes U(1)$  (sec.12.8) as in the MSM, which requires a colour octet of vector superfields  $V^a$ , a weak triplet  $V^{(\delta)}$  and a hypercharge singlet  $V$ . Thus, the kinetic terms of all the fields in eq.(11.17) now fixed by gauge invariance

$$L = \int d^4\theta \widetilde{\Phi}_{ch}^+ \left( e^{g_1 V T + g_2 V^{(\delta)} T^{(\delta)} + g_3 V^a T^a} \right) \widetilde{\Phi}_{ch} + \left[ \int d^2\theta \frac{1}{4} \left( W W + W^{(\delta)} W^{(\delta)} + W^a W^a \right) + \text{h.c.} \right], \quad (14.1)$$

where  $\widetilde{\Phi}_{ch}$  is the matter superfields,  $T, T^{(\delta)}, T^a$  are the generators of appropriate representations of the gauge group. The superpotential determines the scalar potential

$$V(A, A^*) = \frac{1}{2} g_1^2 D^2 + \frac{1}{2} g_2^2 D^{\delta 2} + \frac{1}{2} g_3^2 D^{a2} + |P|^2, \quad (14.2)$$

where the functions  $D$  and  $P$  are given in eq.(11.17).

- By the index  $I = 1, 2, 3$  in the MW-SUSY Lagrangian eq.(11.35) will be labeled the three families of chiral quarks  $q_L^I, u_R^I, d_R^I$ , and chiral leptons  $l_L^I, e_R^I$ , where all of them are Weyl fermions. A SUSY requires the presence of supersymmetric partners which form supermultiplets with known particles, i.e., for every field of SM there is a superpartner with the exact same gauge quantum numbers. Then, the quarks and leptons are promoted to chiral multiplets by adding scalar (spin-0) squarks ( $\tilde{q}_L^I, \tilde{u}_R^I, \tilde{d}_R^I$ ) and sleptons ( $\tilde{l}_L^I, \tilde{e}_R^I$ ) to the spectrum. The gauge bosons are promoted to vector supermultiplet by adding their SUSY partners gauginos (spin- $\frac{1}{2}$ ) ( $\tilde{G}, \tilde{W}, \tilde{B}$ ) to the spectrum.

A content of superfields of MSMSM presents in Table 1:

	supermultiplet	$F$ $B$	$SU_c(3)$ $SU(2)_L$ $U(1)_Y$	$U(1)_{em}$
quarks	$Q_L^I = \begin{pmatrix} U_L^I \\ D_L^I \end{pmatrix}$	$q_L^I$ $\tilde{q}_L^I$	3   2   1/6	$\begin{pmatrix} 2/3 \\ -1/3 \end{pmatrix}$
	$U_R^I$ $D_R^I$	$u_R^I$ $\tilde{u}_R^I$ $d_R^I$ $\tilde{d}_R^I$	$\bar{3}$ 1 $-2/3$ $\bar{3}$ 1   1/3	$-2/3$ 1/3
leptons	$L_L^I = \begin{pmatrix} \mathcal{N}_L^I \\ E_L^I \end{pmatrix}$	$l_L^I$ $\tilde{l}_L^I$	1   2 $-1/2$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
	$E_R^I$	$e_R^I$ $\tilde{e}_R^I$	1   1   1	1
Higgs	$H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$	$\begin{pmatrix} \tilde{h}^0 \\ \tilde{h}^- \end{pmatrix}$ $\begin{pmatrix} h_d^0 \\ h_d^- \end{pmatrix}$	1   2 $-1/2$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
	$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}$	$\begin{pmatrix} \tilde{h}^+ \\ \tilde{h}^0 \end{pmatrix}$ $\begin{pmatrix} h_u^+ \\ h_u^0 \end{pmatrix}$	1   2   1/2	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
gauge bosons	G	$\tilde{G}$ $G$	8   1   0	0
	W	$\tilde{W}$ $W$	1   3   0	$(0, \pm 1)$
	B	$\tilde{B}$ $B$	1   1   0	0

Table 1. Field content of MSMSM. The column below F(B) denotes its fermionic (bosonic) content.

Once the field content is fixed, putting it all together the most generic renormalizable MW-SUSY Lagrangian of MSMSM defined on the SMM:  $SG_N$  eq.(9.1) ensues from the eq.(11.35), which is now invariant under local gauge symmetry  $SU_c(3) \otimes SU(2)_L \otimes U(1)$ , where a set of gauge fields are coupled to various superfields among which is also Higgs supermultiplets. Furthermore, we especially separated from the rest the piece containing only the  $\eta$ -components of the particles defined on the supermanifold  $SG_\eta$ , which, according to sec.9, is important for the further discussion of a realistic realization of the MSMSM (next sec.). To facilitate writing we shall forbear here to write out the piece of Lagrangian containing only the terms of sparticles, as it is a somewhat lengthy and so standard. But, in the mean time, we shall retain the explicit terms of Higgs bosons arisen in the internal  $W$ -world to emphasize the specific mechanism of the electroweak symmetry breaking

discussed in the subsec.12.14. The resulting Lagrangian reads

$$\begin{aligned}
L_{SG_N} = & -\frac{1}{4} \sum_{(a)=1}^3 \left( \left( F_{\eta}^b{}_{mn} F_{\eta}^{mn}{}^b \right)_{(a)} \right) - D_W^m H_u D_W^m H_u^* - D_W^m H_d D_W^m H_d^* \\
& + \sum_{I=1}^3 \left( -i \bar{q}_L^I \sigma_{\eta}^m D_{\eta}^m q_L^I - i \bar{u}_R^I \sigma_{\eta}^m D_{\eta}^m u_R^I - i \bar{d}_R^I \sigma_{\eta}^m D_{\eta}^m d_R^I - i \bar{l}_L^I \sigma_{\eta}^m D_{\eta}^m l_L^I \right. \\
& \left. - i \bar{e}_R^I \sigma_{\eta}^m D_{\eta}^m e_R^I \right) - \sum_{I,J=1}^3 \left( (Y_u)_{IJ} (H_u^* + H_d^*) q_L^I u_R^J + (Y_d)_{IJ} (H_u + H_d) q_L^I d_R^J \right. \\
& + (Y_l)_{IJ} (H_u + H_d) l_L^I e_R^J + \text{h.c.} \left. \right) + \sum_{(a)=1}^3 i g \sqrt{2} \left( H_u^* T^b \widetilde{H}_u \lambda^b - \bar{\lambda}^b \widetilde{H}_u^* T^b H_u \right. \\
& + H_d^* T^b \widetilde{H}_d \lambda^b - \bar{\lambda}^b \widetilde{H}_d^* T^b H_d + H_u^* T^b \widetilde{H}_d \lambda^b + H_d^* T^b \widetilde{H}_u \lambda^b - \bar{\lambda}^b \widetilde{H}_u^* T^b H_d \\
& \left. - \bar{\lambda}^b \widetilde{H}_d^* T^b H_u \right)_{(a)} + \sum_{(a)=1}^3 i g \sqrt{2} \left( A_J^* T_I^b \chi^I \lambda^b - \bar{\lambda}^b \bar{\chi}^I T_J^b A^J \right) - V_u(H_u, H_u^*) \\
& - V_d(H_d, H_d^*) - \frac{1}{8} (g^2 + g'^2) (|H_u|^2 - |H_d|^2)^2 + \frac{1}{2} g^2 |H_u H_d^*|^2 \\
& + \text{all the terms containing only the sparticles.}
\end{aligned} \tag{14.3}$$

Here  $\chi^I$  runs over all the particles, while  $A^J$  runs over all the sparticles, the index  $(a)$  labels the 3 different features in the gauge group,  $V_d(H, H^*)$  is the scalar potential for each Higgs doublet

$$V_u(H_u, H_u^*) = -\frac{1}{2} m_u^2 |H_u|^2 + \frac{1}{4} \lambda_u^2 |H_u|^4 \quad V_d(H_d, H_d^*) = -\frac{1}{2} m_d^2 |H_d|^2 + \frac{1}{4} \lambda_d^2 |H_d|^4. \tag{14.4}$$

A contribution of the “ $D$ ” term to the Higgs potential has also taken into account in last term in eq.(14.3)

$$V_D = \frac{1}{2} D^{(a)} D^{(a)}, \quad D^{(a)} = -g A^{I*} T_{IJ}^a A^J, \tag{14.5}$$

or

$$V_D = \frac{g^2 + g'^2}{8} (|H_u|^2 - |H_d|^2)^2 + \frac{1}{2} g^2 |H_u H_d^*|^2. \tag{14.6}$$

The number of major free parameters in the Lagrangian eq.(14.3) are the primary coupling constants  $\lambda_Q$  and  $\lambda_W$  of nonlinear fermion interaction of the internal MW-components  $i = Q, W$  and gauge couplings  $g_1 g_2 g_3$ . The SM relation  $Q_e = g_1 \cos \theta_W$  holds, where  $\theta_W$  is the weak mixing angle  $\cos^2 \theta_W = g^2 / (g^2 + g'^2)$  (sec.12.8). The Yukawa couplings  $(Y_l Y_l')$  are given in eq.(12.9.2):  $Y = f_Q = Z_Q$ .

## 15 Realistic realization of the MW-SUSY: M\$MSM

The MW-SUSY cannot be an exact symmetry of nature and has to be realized in its broken phase (sec.9). The major point of our strategy is a realistic realization of the supersymmetric extension of the MSM. Thus, the test of the theory will depend on its ability to account for the breaking of the MW-SUSY as well. Here, suggested approach creates a particular incentive for its study. In previous sections we have made a headway of reasonable framework of exact MW-SUSY defined on the exact MW-supermanifold  $SG_N$ . Therefore, one will be able to verify its virtues manifested, first of all, in the

power of boson-fermion cancellations. One of the two principal offshoots of the supersymmetrization of the MSM is the solution of the zero point energy problem. Also, in its unbroken form it solves the technical aspects of the naturalness and hierarchy problem (sec.1), when in non-SUSY theories scalar fields receive large mass corrections even if the bare mass is set to zero, and small masses are “unnatural” [124-126]. This applied to the Higgs bosons of the SM (as well as MSM) yields a difficulty in understanding of the smallness of  $M_Z$  and how it can be kept stable against quantum corrections in some extensions of the SM containing apart from the weak scale  $M_Z$  also a second larger scale  $M_{GUT} \gg M_Z$  [126,127], which holds in Grand Unified theories. The cancellation of quadratic divergences in SUSY theories is a consequence of general non-renormalization theorem [128,129] or the “taming” of the quantum corrections, which stabilizes the Higgs mass and thus weak scale  $M_Z$  without fine-tuning. It is remarkable that these attractive features of the unbroken MSMSM can be maintained as well in the broken M\$MSM. Achieving it one should perform an inverse passage ( $SG \xrightarrow{\eta} G$ ) to the \$MM:  $\$G_N$  eq.(8.5). It is due to the fact that the most powerful boson-fermion cancellation can be regarded as a direct consequence of a constraint stemming from holomorphy, therefore, it should be held even in the M\$MSM. Then the Lagrangian  $L_{\$G_N}$  of the M\$MSM ensues from the Lagrangian  $L_{SG_N}$  eq.(14.3) of the MSMSM (eq.(9.11)-eq.(9.13))

$$L_{\$G_N} = L_{SG_N} + L_{soft}, \quad (15.1)$$

where, according to the sec.9, one has

$$L_{soft} = \left( -m_{IJ}^2 A^I A^J - \frac{1}{2} \widetilde{m}_{ab} \lambda^a \lambda^a - \frac{1}{2} m_u \widetilde{H}_u \widetilde{H}_u - \frac{1}{2} m_d \widetilde{H}_d \widetilde{H}_d + \text{h.c.} \right) + b \epsilon_{ij} \left( H_u^i H_d^j + \text{h.c.} \right). \quad (15.2)$$

Here  $m_{IJ}^2$  is the mass matrix for all the scalars of the chiral multiplets,  $m \equiv (\widetilde{m}_{ab}, m_u, m_d)$  is the mass matrix respectively for the gauginos of each factor of the gauge group, and Higgsinos. The last term of the interaction is induced for the following reason: according to eq.(14.4) and subsec.12.16, these doublets above in free states imply

$$\Delta \hat{m}_u^2 = -m_u^2 + \lambda_u^2 v_u^2 = 0, \quad \Delta \hat{m}_d^2 = -m_d^2 + \lambda_d^2 v_d^2 = 0, \quad (15.3)$$

where  $m^2, \lambda^2, v^2$  are respectively the mass, the coupling constant and VEV of given doublet. In the case at hand, certainly, there is an interaction between the bosons  $H_u$  and  $H_d$  described by the last term in eq.(15.2), when the strength of interaction  $b$  will be fixed through the minimization conditions of the total Higgs potential. This can be used to derive a more physical relationship among the physical parameters. As it will be seen in sec.16 the case

$$\Delta \hat{m}_u^2 = -\Delta \hat{m}_d^2 \neq 0 \quad (15.4)$$

corresponds to the situation when the axion  $A^0$  ( $m_{A^0}^2 = 0$ ) has arisen after the breaking of electroweak gauge symmetry. But the other case of

$$(\hat{m}_u^2 > 0, \quad \hat{m}_d^2 > 0) \quad \text{or} \quad (\hat{m}_u^2 < 0, \quad \hat{m}_d^2 < 0), \quad (15.5)$$

implies an existence of the neutral physical particle of the mass

$$m_{A^0}^2 = \hat{m}_u^2 + \hat{m}_d^2 \neq 0. \quad (15.6)$$

Note that such Higgs doublets arisen on equal footing have counted off from the same point of origin for the same vacuum eq.(12.13.5), then we will be interested physically in the most important simplest case when the electroweak symmetry breaking is parametrized just only by the single Higgs VEV

$$v_u = v_d, \quad \hat{m}_u^2 = \hat{m}_d^2 > 0. \quad (15.7)$$

Of course, we shall carry out a computation in the generic case of eq.(15.6), but in the aftermath we shall turn to the case of eq.(15.7). The non-supersymmetric breaking terms do not spoil a condition of cancellation of quadratic divergences, i.e., a mass-squared sum rule [94]

$$Str M^2 \equiv \sum_{J=0}^1 (-1)^{2J} (2J+1) Tr M_J^2 = const. \quad (15.8)$$

where  $\vec{J}$  is the spin of the particles. It holds independently of the values of the scalar fields. Eventually the mass terms for the scalars contribute a constant, field independent piece in eq.(15.8), while a generic mass matrix for the fermions reads

$$M_{1/2} = M_{1/2}^S + \delta M_{1/2}, \quad (15.9)$$

where  $M_{1/2}^S$  is the supersymmetric part of  $M_{1/2}$ , when  $\delta M_{1/2}$  is given

$$\delta M_{1/2} = \begin{pmatrix} \delta P_{IJ} & \delta D_I^b \\ \delta D_J^a & \delta \widetilde{m} \end{pmatrix}. \quad (15.10)$$

A computation for the considered fields gives  $\delta P = 0 = \delta D$ , while  $\delta \widetilde{m}$  can be arbitrary.

## 16 The generating functional of the viable microscopic theory of the standard model (VMSM)

In previous sections we have systematically build up the M\$MSM. In what follows we shall be motivated by the purpose to derive a final generating functional of the VMSM defined on the four-dimensional Minkowski space  $M_4$ , which is free of all the problems of MSM. The relevant steps are as follows: we start with generic functional  $Z_{real}[\mathcal{J}]$  eq.(9.11) exploring the Lagrangian eq.(15.1), then we extract the pertinent piece containing only

the  $\eta$ -field components defined on  $G_\eta$  :  $\left(Ext_{G_\eta}^G\right)$ . In the aftermath, by passing to  $M_4$ ,  $\left(G_\eta \rightarrow M_4\right)$  (sec.2.1) we get

$$\begin{aligned} Z_{VMSM}[\mathcal{J}] &\equiv Z_{M_4}[\mathcal{J}] = Z_{G_\eta}[\mathcal{J}] \left(G_\eta \rightarrow M_4\right) \equiv Ext_{G_\eta}^G(Z_{real}[\mathcal{J}]) \left(G_\eta \rightarrow M_4\right) \\ &= \left( \int \mathcal{D}[\varphi] \exp \left\{ i \int d^4x (L_{VMSM} + \mathcal{J} \varphi) \right\} \right) \left( R; \underset{x}{A}_{sp} \rightarrow 1 \right), \end{aligned} \quad (16.1)$$

where, as before, with retained terms of Higgs bosons arisen in the  $W$ -world (subsec.12.14), put forth only in illustration of a point at issue, the Lagrangian of the VMSM can be

written down

$$\begin{aligned}
L_{VMSM} = & -\frac{1}{4} \sum_{(a)=1}^3 \left( \left( F_x^b{}_{mn} F_x^{mnb} \right)_{(a)} \right) - \bar{D}_m H_u \bar{D}^m H_u^* - \bar{D}_m H_d \bar{D}^m H_d^* \\
& + \sum_{I=1}^3 \left( -i \bar{q}_L^I \not{D} q_L^I - i \bar{u}_R^I \not{D} u_R^I - i \bar{d}_R^I \not{D} d_R^I - i \bar{l}_L^I \not{D} l_L^I \right. \\
& \left. - i \bar{e}_R^I \not{D} r_R^I \right) - \sum_{I,J=1}^3 \left( (Y_u)_{IJ} (H_u^* + H_d^*) q_L^I u_R^J + (Y_d)_{IJ} (H_u + H_d) q_L^I d_R^J \right. \\
& + (Y_l)_{IJ} (H_u + H_d) l_L^I e_R^J + \text{h.c.} \left. \right) + \left[ \sum_{(a)=1}^3 i g \sqrt{2} \left( H_u^* T^b \widetilde{H}_u \lambda^b - \bar{\lambda}^b \widetilde{H}_u^* T^b H_u \right. \right. \\
& + H_d^* T^b \widetilde{H}_d \lambda^b - \bar{\lambda}^b \widetilde{H}_d^* T^b H_d + H_u^* T^b \widetilde{H}_d \lambda^b + H_d^* T^b \widetilde{H}_u \lambda^b - \bar{\lambda}^b \widetilde{H}_u^* T^b H_d \\
& \left. \left. - \bar{\lambda}^b \widetilde{H}_d^* T^b H_u + A_J^* T_I^{bJ} \chi^I \lambda^b - \bar{\lambda}^b \bar{\chi}^I T_J^{bI} A^J \right)_{(a)} \right]_{loop} - V_u(H, H^*), \tag{16.2}
\end{aligned}$$

where  $\not{D} = \sigma^m \bar{D}_m$ , and the scalar potential reads

$$\begin{aligned}
V(H, H^*) = & V_u(H_u, H_u^*) + V_d(H_d, H_d^*) + b (\epsilon_{ij} H_u^i H_d^j + \text{h.c.}) \\
& + \frac{1}{8} (g^2 + g'^2) (|H_u|^2 - |H_d|^2)^2 + \frac{g^2}{2} |H_u H_d^*|^2. \tag{16.3}
\end{aligned}$$

The magnitudes of the quartic potential terms are not arbitrary, i.e., they are constrained by the supersymmetry to be of magnitude  $g^2$  and  $g'^2$ . Since at the limit  $A_{sp} \rightarrow 1$  in eq.(16.1) all the sparticles become independent of the  $x$ -field component as well of  $x$ -coordinate ( $x \in M_4$ ), therefore the sparticles  $A^\pm$  involved in the brackets may finally contribute only in the radiative loop corrections to the fermion and boson mass of all the particles maintaining the boson-fermion cancellations. Here, as before,  $\chi^\pm$  runs over all the particles, while  $A^\pm$  runs over all the sparticles. In the VMSM electroweak symmetry breaking slightly complicated due to the two complex Higgs doublets instead of just one in the MSM. This completes the definition of the VMSM. We are now ready to investigate some of its properties in more details. It is reasonable to start discussion of the phenomenology of VMSM with a treatment of its Higgs sector. As before, the  $SU(2)_L \otimes U(1)_Y$  should be broken spontaneously down to electromagnetism  $U(1)_{em}$  simultaneously in both the  $W$ - world and  $M_4$  (subsec.12.14), for which the scalar potential eq.(16.3) should have its absolute minimum away from the origin. Using the  $SU(2)$  gauge transformations we can ignore the charged components without loss of generality when minimizing the potential and establish the conditions necessary for  $h_u^0$  and  $h_d^0$  to get nonzero VEVs. Furthermore, they can be chosen to be real:  $\langle h_u^0 \rangle = v_u$ , and  $\langle h_d^0 \rangle = v_d$  while  $\langle h_u^\pm \rangle = \langle h_d^\pm \rangle = 0$ . In standard manners these VEVs can be connected to the known mass of the  $Z^0$  boson and the electroweak gauge couplings:

$$M_Z^2 = \frac{1}{2} (g^2 + g'^2) v^2 \equiv \frac{1}{2} \tilde{g}^2 v^2, \tag{16.4}$$

where  $v_u = v \sin \beta$ ,  $v = v \cos \beta$ . The VEVs are directly related to the parameters of the Higgs potential

$$\begin{aligned}
V = & -\frac{1}{2} m_u^2 \left( |h_u^0|^2 + |h_u^+|^2 \right) + \frac{1}{4} \lambda_u^2 \left( |h_u^0|^2 + |h_u^+|^2 \right)^2 - \frac{1}{2} m_d^2 \left( |h_d^0|^2 + |h_d^+|^2 \right) \\
& + \frac{1}{4} \lambda_d^2 \left( |h_d^0|^2 + |h_d^+|^2 \right)^2 + \left( b \left( h_u^+ h_d^- - h_u^0 h_d^0 \right) + \text{h.c.} \right) + \frac{1}{4} \frac{M_Z^2}{v^2} \left( |h_u^0|^2 + |h_u^+|^2 \right. \\
& \left. - |h_d^0|^2 - |h_d^-|^2 \right)^2 + \frac{M_W^2}{v^2} |h_u^+ h_d^{0*} + h_u^0 h_d^{-*}|^2
\end{aligned} \tag{16.5}$$

via the minimization condition

$$\begin{aligned}
\frac{\partial V}{\partial h_u^0} &= \left( -m_u^2 + \lambda_u^2 v_u^2 - M_Z^2 \cos 2\beta \right) v_u - 2 b v_d = 0, \\
\frac{\partial V}{\partial h_d^0} &= \left( -m_d^2 + \lambda_d^2 v_d^2 + M_Z^2 \cos 2\beta \right) v_d - 2 b v_u = 0,
\end{aligned} \tag{16.6}$$

or

$$\hat{m}_u^2 = b \cot \beta + \frac{1}{2} M_Z^2 \cos 2\beta, \quad \hat{m}_d^2 = b \tan \beta - \frac{1}{2} M_Z^2 \cos 2\beta, \tag{16.7}$$

provided by (eq.(12.12.7))

$$\begin{aligned}
m_u^2 &= \frac{24}{7 \zeta(3)} \left( \frac{\hbar}{\xi_0} \right)^2 \left( \frac{\mu_u}{m} \right)^2 \left[ 1 - \frac{T}{T_{c\mu}} - \left( \frac{\mu_u}{m} - 1 \right) \ln \frac{2\tilde{\omega}}{\Delta_0} \right], \\
m_d^2 &= \frac{24}{7 \zeta(3)} \left( \frac{\hbar}{\xi_0} \right)^2 \left( \frac{\mu_d}{m} \right)^2 \left[ 1 - \frac{T}{T_{c\mu}} - \left( \frac{\mu_d}{m} - 1 \right) \ln \frac{2\tilde{\omega}}{\Delta_0} \right], \\
\lambda_u^2 &= \frac{96}{7 \zeta(3)} \left( \frac{\hbar}{\xi_0} \right)^2 \frac{1}{N_u}, \quad \lambda_d^2 = \frac{96}{7 \zeta(3)} \left( \frac{\hbar}{\xi_0} \right)^2 \frac{1}{N_d}.
\end{aligned} \tag{16.8}$$

Here the Higgs mechanism works in the following way: Before the symmetry was broken in the  $W$ -world, the 2 complex  $SU(2)_L$  Higgs doublets had 8 degrees of freedom. Three of them were the would-be Nambu-Goldstone bosons  $G^0, G^\pm$ , which were absorbed to give rise the longitudinal modes of the massive  $W$ -components of the  $Z^0$  and  $W^\pm$  vector bosons, which simultaneously give rise the corresponding  $x$ - components too, leaving 5 physical degrees of freedom. The latter consists of a charged Higgs boson pairs  $H^\pm$ , a CP-odd neutral Higgs boson  $A^0$ , and CP-even neutral Higgs bosons  $h^0$  and  $H^0$ . The mass eigenstates and would-be Nambu-Goldstone bosons are made of the original gauge-eigenstate fields, where the physical pseudoscalar Higgs boson  $A^0$  is made of from the imaginary parts of  $h_u^0$  and  $h_d^0$ , and is orthogonal to  $G^0$ ; while the neutral scalar Higgs bosons are mixtures of the real parts of  $h_u^0$  and  $h_d^0$ . The CP conservation in the Higgs sector is automatic [19,23]. The mass of any physical Higgs boson that is SM-like is strictly limited, as are the radiative corrections to the quartic potential terms. The tree-level masses for these Higgs states are calculated from the mass matrices of second



derivatives of the Higgs potential:

$$\begin{aligned}
M_{\mathcal{J}}^2 &= \frac{1}{2} \frac{\partial^2 V}{\partial (\mathcal{I}m[h_i^0]) \partial (\mathcal{I}m[h_j^0])} = \frac{1}{2} m_A^2 \sin 2\beta \begin{pmatrix} \tan \beta & 1 \\ 1 & \cot \beta \end{pmatrix}, \\
M_{\mathcal{R}}^2 &= \frac{1}{2} \frac{\partial^2 V}{\partial (\mathcal{R}e[h_i^0]) \partial (\mathcal{R}e[h_j^0])} = \frac{1}{2} m_A^2 \sin 2\beta \begin{pmatrix} \tan \beta + \tau_d \cot \beta & -1 \\ -1 & \cot \beta + \tau_u \tan \beta \end{pmatrix}, \\
&+ \frac{1}{2} M_Z^2 \sin 2\beta \begin{pmatrix} \cot \beta & -1 \\ -1 & \tan \beta \end{pmatrix}, \\
M_{\pm}^2 &= \frac{1}{2} \frac{\partial^2 V}{\partial h_i^- \partial h_j^+} = \frac{1}{2} M_{H^\pm}^2 \sin 2\beta \begin{pmatrix} \tan \beta & 1 \\ 1 & \cot \beta \end{pmatrix},
\end{aligned} \tag{16.9}$$

where

$$\begin{aligned}
m_A^2 &= \hat{m}_u^2 + \hat{m}_d^2 = \frac{2b}{\sin 2\beta}, \quad \tau_d = \frac{\lambda_d^2 v^2}{m_A^2}, \quad \tau_u = \frac{\lambda_u^2 v^2}{m_A^2}, \\
M_{H^\pm}^2 &= M_W^2 + m_A^2, \quad i, j = u, d.
\end{aligned} \tag{16.10}$$

Once one has adopted the standard parameters then all the physical observables can be expressed in terms of them. The eigenvalues of  $M_{\mathcal{J}}^2$  are  $m_{G^0}^2 = 0$  and  $m_A^2$ . The  $m_A^2$  and  $\tan \beta$  can be chosen as the theory parameters. The eigenvalues of the  $M_{\mathcal{R}}^2$  are written

$$\begin{aligned}
m_{H^0}^2 &= \frac{1}{2} \left[ m_{A_1}^2 + M_Z^2 + \sqrt{(m_{A_1}^2 + M_Z^2)^2 - 4 m_A^2 M_Z^2 \left( \cos^2 2\beta + \theta_\tau \frac{m_A^2}{M_Z^2} \right)} \right], \\
m_{h^0}^2 &= \frac{1}{2} \left[ m_{A_1}^2 + M_Z^2 - \sqrt{(m_{A_1}^2 + M_Z^2)^2 - 4 m_A^2 M_Z^2 \left( \cos^2 2\beta + \theta_\tau \frac{m_A^2}{M_Z^2} \right)} \right],
\end{aligned} \tag{16.11}$$

where

$$\begin{aligned}
m_{A_1}^2 &= m_A^2 \left( 1 + \tau_d \cos^2 \beta + \tau_u \sin^2 \beta \right), \\
\theta_\tau &= \tau_d \cos^4 \beta + \tau_u \sin^4 \beta + \frac{1}{4} \sin^2 2\beta \left[ \tau_d \tau_u + (\tau_d + \tau_u) \frac{M_Z^2}{m_A^2} \right].
\end{aligned} \tag{16.12}$$

It is then easy by ordinary manipulations to investigate the region of definition of the  $m_{h^0}^2$  in several cases. But, as it is mentioned in previous section we are interested in the physically most important case when the electroweak symmetry breaking is parametrized just only by the single Higgs VEV  $v_u = v_d = \frac{v}{\sqrt{2}}$  and  $\hat{m}_u^2 = \hat{m}_d^2 > 0$  (eq.(15.7)), namely

$$\begin{aligned}
b &= \hat{m}_u^2 = \hat{m}_d^2 > 0, \quad m_A^2 = 2b > 0, \\
\lambda_u^2 v_u^2 &= \frac{m_A^2}{2} + m_u^2, \quad \lambda_d^2 v_d^2 = \frac{m_A^2}{2} + m_d^2.
\end{aligned} \tag{16.13}$$

Hence, the  $M_{\mathcal{R}}^2$  is written down

$$M_{\mathcal{R}}^2 = \begin{pmatrix} M^2 + m_d^2 & -M^2 \\ -M^2 & M^2 + m_u^2 \end{pmatrix}, \tag{16.14}$$

where  $M^2 = \frac{1}{2} (M_Z^2 + m_A^2)$ . The eigenvalues of the  $M_{\mathcal{R}}^2$  then will be

$$\begin{aligned} m_{H^0}^2 &= \frac{1}{2} \left[ m_u^2 + m_d^2 + 2 M^2 + \sqrt{(m_u^2 - m_d^2)^2 + 4 M^4} \right], \\ m_{h^0}^2 &= \frac{1}{2} \left[ m_u^2 + m_d^2 + 2 M^2 - \sqrt{(m_u^2 - m_d^2)^2 + 4 M^4} \right]. \end{aligned} \quad (16.15)$$

In the limit  $|m_u^2 - m_d^2| \ll 2 M^2$  it gives

$$m_{h^0}^2 \simeq \frac{1}{2} (m_u^2 + m_d^2)^2, \quad m_{H^0}^2 \simeq m_{h^0}^2 + M_Z^2, \quad (16.16)$$

while at  $|m_u^2 - m_d^2| \gg 2 M^2$  one has

$$m_{H^0}^2 \simeq m_u^2 + m_A^2 + M_Z^2, \quad m_{h^0}^2 \simeq m_d^2 + m_A^2 + M_Z^2. \quad (16.17)$$

In generic, if  $|m_u^2 - m_d^2| \leq 2 M^2$  then the lower bound of  $m_{h^0}^2$  can be written

$$m_{h^0}^2 \geq \frac{1}{2} \left[ m_u^2 + m_d^2 - (\sqrt{2} - 1) (m_A^2 + M_Z^2) \right], \quad (16.18)$$

and if  $|m_u^2 - m_d^2| \geq 2 M^2$ , then

$$m_{h^0}^2 \geq \frac{1}{2} \left[ m_A^2 + M_Z^2 + m_d^2 (1 + \sqrt{2}) - m_u^2 (\sqrt{2} - 1) \right]. \quad (16.19)$$

In the same manner one gets the eigenvalues of the  $M_{\pm}^2$

$$m_{G^{\pm}}^2 = 0, \quad m_{H^{\pm}}^2 = M_W^2 + m_A^2. \quad (16.20)$$

It is important to mention that unlike the conventional MSSM models a suggested VMSM does not predict the existence of any light neutral Higgs boson  $h^0$  in the supersymmetric two-doublets Higgs sector. In contrary, both of the  $H^0$  and  $h^0$  bosons have enough large masses, which is straightforward to see from the estimates eq.(16.15)-eq.(16.19). Especially in the case of particular interest  $m_u^2 \sim m_d^2 \gg M_Z^2$ , the  $H^0$  and  $h^0$  are much heavier than the  $Z^0$  boson. Furthermore, it is well known fact for the Higgs bosons that the one-loop radiative corrections for some of their masses can push up the upper bound even significantly.

## 17 Three solid testable predictions of VMSM

Discussing now the relevance of our present approach to the physical realities we should attempt to provide some ground for checking the predictions of the VMSM against experimental evidence. It is remarkable that the resulting theory makes plausible following three testable implications for the current experiments at LEP2, at the Tevatron and LHC discussed below, which are drastically different from the predictions of conventional models:

1. At first recall that, in conventional scenario there is a great belief for the  $h^0$  boson having the mass  $m_{h^0} \leq M_Z$ , which would be the only Higgs boson that can be discovered

at the next round of colliders. Therefore, searches for this boson (if the mass is below 150 GeV or so [39]) would be one of the crucial points in testing of MSSM models in particular, as well of conventional SUSY in general. This prediction remains one of characteristic features of such theories and it even holds in the limit that all masses of the supersymmetric particles are sent to infinity when one recovers the non-supersymmetric Standard Model. In the same time searches for this boson will invalidate the whole framework of VMSM or will serve as the direct indication of its validity, where, in contrast to the formers, we are led to reject such an expectation due to the specific mechanism of the electroweak symmetry breaking. While the two important phenomenological implication of the MSM given in the subsec.12.14 just are the first two testable predictions of the VMSM for the current experiments.

2. It is well known that once  $SU(2)_L \otimes U(1)_Y$  is broken, the fields with different  $SU(2)_L \otimes U(1)_Y$  quantum numbers can mix if they have the same  $SU(3)_c \otimes U(1)_{em}$  quantum numbers. Such a phenomenon occurs in the sfermion sector of the M\$MSM. If one ignore mixing between sfermions of different generations but will include the mixing between  $SU(2)$  doublet and singlet sfermions then the sfermion mass matrix decomposes into a series of  $2 \times 2$  matrices of the sfermions of a given flavour. The charginos are mixtures of the charged Higgsinos and the charged gauginos, and neutralinos are the mixture of neutral Higgsinos and the neutral gauginos, etc.. We can readily obtain the resulting explicit forms of corresponding mass matrices within standard technique. But shall forbear to write them out here as the sfermions are no longer of consequence for discussion of the final fields defined on  $M_4$ . The sparticles could never emerge in  $M_4$  (eq.16.1) and will be of no interest for the future experiments. By this we arrived to the second principle point of drastic deviation of M\$MSM from the conventional MSSM models. In MSSM models as well in any conventional SUSY theory the supersymmetry was implemented in the Minkowski space  $M_4$  by adding a new four odd dimensions, and there are two major motivation for SUSY to be realized in the TeV range, i.e., the masses of sparticles are of the order of a few TeV or less. First one is a solution of the hierarchy problem, when in order to introduce no new fine-tuning all soft terms should be of the same order of magnitude at most in the TeV range-weak scale (e.g. [65]). The second motivation for low energy SUSY comes from the view point of gauge unification (a supersymmetric GUT). Since the current experiments at LEP2, at the Tevatron and at LHC will explore this energy range, then, the second great expectation of such theories arise that at least some of the sparticles can be found and their parameters like masses and coupling constants will also be measured (the precise measurements). Reflecting upon the results far obtained here, in a strong contrast to such theories the unbroken MW-SUSY is implemented on the MW-SMM:  $SG_N$  by, at first, lifting up  $G \rightarrow SG_\eta$  and consequently making an inverse passage to the  $\$G_N$  ( $SG_\eta \rightarrow G_\eta$ ) on which the resulting theory M\$MSM is defined (sec.16). Applying the final passage ( $G_\eta \rightarrow M_4$ ) (eq.(16.1)) we arrive to the final VMSM, where only the particles will survive on the  $M_4$  at the real physical limit under the R-parity conservation (eq.(16.1)). Then,

- *all the sparticles never could emerge in the  $M_4$  neither at TeV range nor at any energy range at all.*

From the view point of achieving the final potentially realistic supersymmetric field theory this will be third crucial test in experiments above for verifying the efforts made either in MSSM model building (the conventional SUSY theories) or in suggested VMSM (the

MW-SUSY), which are based on two quite different approaches. To sum up the discussion thus far, we have argued that, in strong contrast to conventional SUSY theories, if the VMSM given here proves viable it becomes an crucial issue to hold in experiments the above-mentioned three tests.

## 17.1 Quark flavour mixing and the Cabibbo angles

An implication of quark generations into general scheme will be carried out in the same way of the leptons (subsec.12). But before proceeding further that it is profitable to enlarge it by the additional assumption without asking the reason behind it:

- The MW components imply

$${}^i\bar{\psi}_u^A(\dots, \theta_{i_1}, \dots, \theta_{i_n}, \dots) {}^j\psi_u^B(\dots, \theta_{i_1}, \dots, \theta_{i_n}, \dots) = \delta_{ij} \sum_{l=i_1, \dots, i_n} f_{il}^{AB} {}^i(\bar{q}_l q_l), \quad (17.1.1)$$

namely, the contribution of each individual subquark  ${}^i q_l$ , into the component of given world ( $i$ ) is determined by the *partial formfactor*  $f_{il}^{AB}$ . Under the group  $SU(2) \otimes U(1)$  the left-handed quarks transform as three doublets, while the right-handed quarks transform as independent singlets except of following differences:

1. The values of weak-hypercharge of quarks are changed due to their fractional electric charges  $q_L : Y^w = \frac{1}{3}$ ,  $u_R : Y^w = \frac{4}{3}$ ,  $d_R : Y^w = -\frac{2}{3}$  etc.
2. All Yukawa coupling constants have nonzero values.
3. An appearance of quark mixing and Cabibbo angle, which is unknown in the scope of standard model.
4. An existence of CP-violating phase in unitary matrix of quark mixing. We shall discuss it in the next section.

In [2] we attempt to give an explanation to quark mixing and Cabibbo angle. We consider this problem, for simplicity, on the example of four quarks  $u, d, s, c$ . The further implication of all quarks would complicate the problem only in algebraic sense. Instead of mixing of the  $d'$  and  $s'$  it is convenient to consider a quite equivalent mixing of  $u'$  and  $c'$ . Similar formulas can be worked out for the other mixings. Hence, the nonzero value of Cabibbo angle arises due to nonzero coupling constant  $f_{u'c'}$ . The problem is to calculate all coupling constants  $f_{u'c'}, f_{c't'}$ , and  $f_{t'u'}$  generating three Cabibbo angles

$$\tan 2\theta_3 = \frac{2f_{u'c'}}{f_{c'} - f_{u'}}, \quad \tan 2\theta_1 = \frac{2f_{c't'}}{f_{t'} - f_{c'}}, \quad \tan 2\theta_2 = \frac{2f_{t'u'}}{f_{u'} - f_{t'}}.$$

Since the Q-components of the quark fields  $u', c'$  and  $t'$  contain at least one identical subquark, the partial formfactors  $\bar{f}_i$ , as well then all coupling constants, acquire nonzero values causing a quark mixing with the Cabibbo angles ([2]). The resulting expressions are as follows:

$$\begin{aligned} \tan 2\theta_3 &= \frac{\bar{f}_3 (\bar{\Sigma}_{Q_u}^2 + \bar{\Sigma}_{Q_u}^3 + \bar{\Sigma}_{Q_c}^3 + \bar{\Sigma}_{Q_c}^1)}{(\bar{\Sigma}_{Q_c}^3 + \bar{\Sigma}_{Q_c}^1 - \bar{\Sigma}_{Q_u}^2 - \bar{\Sigma}_{Q_u}^3)}, & \tan 2\theta_1 &= \frac{\bar{f}_1 (\bar{\Sigma}_{Q_c}^3 + \bar{\Sigma}_{Q_c}^1 + \bar{\Sigma}_{Q_t}^1 + \bar{\Sigma}_{Q_t}^2)}{(\bar{\Sigma}_{Q_t}^1 + \bar{\Sigma}_{Q_t}^2 - \bar{\Sigma}_{Q_c}^3 - \bar{\Sigma}_{Q_c}^1)}, \\ \tan 2\theta_2 &= \frac{\bar{f}_2 (\bar{\Sigma}_{Q_t}^1 + \bar{\Sigma}_{Q_t}^2 + \bar{\Sigma}_{Q_u}^2 + \bar{\Sigma}_{Q_u}^3)}{(\bar{\Sigma}_{Q_u}^2 + \bar{\Sigma}_{Q_u}^3 - \bar{\Sigma}_{Q_t}^1 - \bar{\Sigma}_{Q_t}^2)}. \end{aligned} \quad (17.1.2)$$

Therefore, the unimodular orthogonal group of global rotations arises, and the quarks  $u', c'$  and  $t'$  come up in doublets  $(u', c'), (c', t')$ , and  $(t', u')$ . For the leptons these formfactors equal zero  $\bar{f}_i^{lept} \equiv 0$ , because of eq.(12.4.1), namely the lepton mixing is absent. In conventional notation  $\begin{pmatrix} u' \\ d \end{pmatrix}_L, \begin{pmatrix} c' \\ s \end{pmatrix}_L, \begin{pmatrix} t' \\ b \end{pmatrix}_L \rightarrow \begin{pmatrix} u \\ d' \end{pmatrix}_L, \begin{pmatrix} c \\ s' \end{pmatrix}_L, \begin{pmatrix} t \\ b' \end{pmatrix}_L$ , which gives rise to  $f_{u'c'} \rightarrow f_{d's'}, f_{c't'} \rightarrow f_{s'b'}, f_{t'u'} \rightarrow f_{b'd'}, f_{u'} \rightarrow f_{d'}, f_{c'} \rightarrow f_{s'}, f_{t'} \rightarrow f_{b'}, f_d \rightarrow f_u, f_s \rightarrow f_c, f_b \rightarrow f_t$ .

## 17.2 The CP-violating phase

The required magnitude of the CP-violating complex parameter  $\varepsilon$  depends upon the specific choice of theoretical model for explaining the  $K_2^0 \rightarrow 2\pi$  decay [130, 131]. From the experimental data it is somewhere  $|\varepsilon| \simeq 2.3 \times 10^{-3}$ . In the framework of Kobayashi-Maskawa (KM) parametrization of unitary matrix of quark mixing [132], this parameter may be expressed in terms of three Eulerian angles of global rotations in the three dimensional quark space and one phase parameter. We attempt to derive the KM-matrix with an explanation given to an appearance of the CP-violating phase ([2]). Recall that during the realization of MW- structure the P-violation compulsory occurred in the W-world provided by the spanning eq.(12.8.1). The three dimensional effective space  $W_v^{loc}(3)$  arises as follows:

$$\begin{aligned} W_v^{loc}(3) \ni q_v^{(3)} &= \begin{pmatrix} q_R^w(\vec{T} = 0) \\ q_L^w(\vec{T} = \frac{1}{2}) \end{pmatrix} \equiv \\ &\equiv \begin{pmatrix} u_R, d_R \\ u' \\ d \end{pmatrix}_L, \begin{pmatrix} c_R, s_R \\ c' \\ s \end{pmatrix}_L, \begin{pmatrix} t_R, b_R \\ t' \\ b \end{pmatrix}_L \equiv \begin{pmatrix} q_3^w \\ q_1^w \\ q_2^w \end{pmatrix}, \begin{pmatrix} q_1^w \\ q_2^w \\ q_3^w \end{pmatrix}, \begin{pmatrix} q_2^w \\ q_3^w \\ q_1^w \end{pmatrix}, \end{aligned} \quad (17.2.1)$$

where the subscript  $(v)$  formally specifies a vertical direction of multiplet, the subquarks  $q_\alpha^w(\alpha = 1, 2, 3)$  associate with the local rotations around corresponding axes of three dimensional effective space  $W_v^{loc}(3)$ . The local gauge transformations  $f_{exp}^v$  are implemented upon the multiplet  $q_v^{(3)} = f_{exp}^v q_v^{(3)}$ , where  $f_{exp}^v \in SU^{loc}(2) \otimes U^{loc}(1)$ . If for the moment we leave it intact and make a closer examination of the content of the middle row in eq.(17.2.1), then we distinguish the other symmetry arisen along the horizontal line  $(h)$ . Hence, we may expect a situation similar to those of subsec.12.8 will be held in present case. The procedure just explained therein can be followed again. We have to realize that due to the specific structure of W-world implying the condition of realization of the MW connections eq.(12.2.5) with  $\vec{T} \neq 0$ ,  $Y^w \neq 0$ , the subquarks  $q_\alpha^w$  tend to be compulsory involved into triplet. They form one “doublet”  $\vec{T} \neq 0$  and one singlet  $Y^w \neq 0$ . Then the quarks  $u'_L, c'_L$  and  $t'_L$  form a  $SO^{gl}(2)$  “doublet” and a  $U^{gl}(1)$  singlet

$$\begin{aligned} ((u'_L, c'_L) t'_L) &\equiv ((q_1^w, q_2^w) q_3^w) \equiv q_h^{(3)} \in W_h^{gl}(3), \\ (u'_L, (c'_L, t'_L)) &\equiv (q_1^w, (q_2^w, q_3^w)), \quad ((t'_L, u'_L) c'_L) \equiv ((q_3^w, q_1^w), q_2^w). \end{aligned} \quad (17.2.2)$$

Here  $W_h^{gl}(3)$  is the three dimensional effective space in which the global rotations occur. They are implemented upon the triplets through the transformation matrix  $f_{exp}^h$ :

$q_h^{(3)} = f_{exp}^h q_h^{(3)}$ , which reads (eq.(17.2.2))

$$f_{exp}^h = \begin{pmatrix} f_{33} & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{pmatrix}$$

in the notation  $c = \cos \theta$ ,  $s = \sin \theta$ . This implies the incompatibility relation eq.(2.5.2), namely

$$\|f_{exp}^h\| = f_{33}(f_{11}f_{22} - f_{12}f_{21}) = f_{33}\varepsilon_{123}\varepsilon_{123}\|f_{exp}^h\|f_{33}^*. \quad (17.2.3)$$

That is  $f_{33}f_{33}^* = 1$ , or  $f_{33} = e^{i\delta}$  and  $\|f_{exp}^h\| = 1$ . The general rotation in  $W_h^{gl}(3)$  is described by Eulerian three angles  $\theta_1, \theta_2, \theta_3$ . If we put the arisen phase only in the physical sector then a final KM-matrix of quark flavour mixing would result. The CP-violating parameter  $\varepsilon$  approximately is written [130]  $\varepsilon \sim s_1 s_2 s_3 \sin \delta \neq 0$ . Thus, while the spanning  $W_v^{loc}(2) \rightarrow W_v^{loc}(3)$  eq.(17.2.1) underlies the P-violation and the expanded symmetry  $G_v^{loc}(3) = SU^{loc}(2) \otimes U^{loc}(1)$ , the CP-violation stems from the similar spanning  $W_h^{gl}(2) \rightarrow W_h^{gl}(3)$  eq.(17.2.2) with the expanded global symmetry group.

### 17.3 The mass-spectrum of leptons and quarks

The mass-spectrum of leptons and quarks stems from their internal MW-structure eq.(12.4.1) and eq.(12.5.1) incorporated with the quark mixing eq.(17.1.2). We start a discussion with the leptons. It is worthwhile to adopt a simple viewpoint on Higgs sector. Following the subsec.12.14, the explicit expressions of the lepton masses read  $m_i = \frac{\eta}{\sqrt{2}}f_i$  and  $m_i^\nu = \frac{\eta}{\sqrt{2}}f_i^\nu$ , that  $m_e : m_\mu : m_\tau = f_e : f_\mu : f_\tau = L_1^2 : L_2^2 : L_3^2$  provided by  $L_1^2 = \frac{m_i}{M}$  and  $\sqrt{M} = \sum_i \sqrt{m_i}$ . Thus,  $L_1 = (8.9; 7.8) \times 10^{-3}$ ,  $L_2 = (0.13; 0.11)$ ,  $L_3 = (0.9; 0.88)$ .

Taking into account the eq.(12.5.1) and eq.(17.1.2) the coupling constants of the quarks  $d, s$  and  $b$  can be written

$$\begin{aligned} f_d &= L_1 \text{tr}(\rho_d \Sigma_Q) \equiv L_1 \tilde{f}_d, & f_s &= L_2 \text{tr}(\rho_s \Sigma_Q) \equiv L_2 \tilde{f}_s, & f_b &= L_3 \text{tr}(\rho_b \Sigma_Q) \equiv L_3 \tilde{f}_b, \\ \rho_d &= \rho^Q \rho_d^B, & \rho_s &= \rho^Q \rho_s^B \rho^s, & \rho_b &= \rho^Q \rho_b^B \rho^b. \end{aligned} \quad (17.3.1)$$

Hence  $m_d = \frac{\eta}{\sqrt{2}}f_d$ ,  $m_s = \frac{\eta}{\sqrt{2}}f_s$ ,  $m_b = \frac{\eta}{\sqrt{2}}f_b$ , and  $m_d : m_s : m_b = (L_1 \tilde{f}_d) : (L_2 \tilde{f}_s) : (L_3 \tilde{f}_b)$ . According to the sec.17, we derive the masses of the  $u, c$  and  $t$  quarks

$$\begin{aligned} m_u &= \frac{\eta}{\sqrt{2}} \left\{ (\bar{\Sigma}_{Q_u}^2 + \bar{\Sigma}_{Q_u}^3) \cos^2 \theta_3 + (\bar{\Sigma}_{Q_c}^3 + \bar{\Sigma}_{Q_c}^1) \sin^2 \theta_3 - \frac{\bar{f}_3}{2} (\bar{\Sigma}_{Q_u}^2 + \bar{\Sigma}_{Q_u}^3 + \right. \\ &\quad \left. \bar{\Sigma}_{Q_c}^3 + \bar{\Sigma}_{Q_c}^1) \sin 2\theta_3 \right\} = \frac{\eta}{\sqrt{2}} \left\{ (\bar{\Sigma}_{Q_u}^2 + \bar{\Sigma}_{Q_u}^3) \cos^2 \theta_2 + (\bar{\Sigma}_{Q_t}^1 + \bar{\Sigma}_{Q_t}^2) \sin^2 \theta_2 + \right. \\ &\quad \left. \frac{\bar{f}_2}{2} (\bar{\Sigma}_{Q_t}^1 + \bar{\Sigma}_{Q_t}^2 + \bar{\Sigma}_{Q_u}^2 + \bar{\Sigma}_{Q_u}^3) \sin 2\theta_2 \right\}, \end{aligned} \quad (17.3.2)$$

$$\begin{aligned} m_c &= \frac{\eta}{\sqrt{2}} \left\{ (\bar{\Sigma}_{Q_u}^2 + \bar{\Sigma}_{Q_u}^3) \sin^2 \theta_3 + (\bar{\Sigma}_{Q_c}^3 + \bar{\Sigma}_{Q_c}^1) \cos^2 \theta_3 + \frac{\bar{f}_3}{2} (\bar{\Sigma}_{Q_u}^2 + \bar{\Sigma}_{Q_u}^3 + \right. \\ &\quad \left. \bar{\Sigma}_{Q_c}^3 + \bar{\Sigma}_{Q_c}^1) \sin 2\theta_3 \right\} = \frac{\eta}{\sqrt{2}} \left\{ (\bar{\Sigma}_{Q_c}^3 + \bar{\Sigma}_{Q_c}^1) \cos^2 \theta_1 + (\bar{\Sigma}_{Q_t}^1 + \bar{\Sigma}_{Q_t}^2) \sin^2 \theta_1 - \right. \\ &\quad \left. \frac{\bar{f}_1}{2} (\bar{\Sigma}_{Q_c}^3 + \bar{\Sigma}_{Q_c}^1 + \bar{\Sigma}_{Q_t}^1 + \bar{\Sigma}_{Q_t}^2) \sin 2\theta_1 \right\}, \end{aligned} \quad (17.3.3)$$

$$\begin{aligned}
m_t = \frac{\eta}{\sqrt{2}} & \left\{ \left( \bar{\Sigma}_{Qt}^1 + \bar{\Sigma}_{Qt}^2 \right) \cos^2 \theta_1 + \left( \bar{\Sigma}_{Qc}^3 + \bar{\Sigma}_{Qc}^1 \right) \sin^2 \theta_1 + \frac{\bar{f}_1}{2} \left( \bar{\Sigma}_{Qt}^1 + \bar{\Sigma}_{Qt}^2 + \right. \right. \\
& \left. \bar{\Sigma}_{Qc}^3 + \bar{\Sigma}_{Qc}^1 \right) \sin 2\theta_1 \Big\} = \frac{\eta}{\sqrt{2}} \left\{ \left( \bar{\Sigma}_{Qt}^1 + \bar{\Sigma}_{Qt}^2 \right) \cos^2 \theta_2 + \left( \bar{\Sigma}_{Qu}^2 + \bar{\Sigma}_{Qu}^3 \right) \sin^2 \theta_2 - \right. \\
& \left. \frac{\bar{f}_2}{2} \left( \bar{\Sigma}_{Qt}^1 + \bar{\Sigma}_{Qt}^2 + \bar{\Sigma}_{Qu}^2 + \bar{\Sigma}_{Qu}^3 \right) \sin 2\theta_2 \right\}. \quad (17.3.4)
\end{aligned}$$

## 18 The physical outlook and concluding remarks

The physical outlook on suggested approach and concluding remarks are given in this section in order to resume once again a whole physical picture and to provide a sufficient background for its understanding without undue hardship. To complete the MSM [1,2], here we attempted to develop its realistic, viable, minimal SUSY extension in order to solve the zero point energy and hierarchy problems standing before it.

- Our scheme based on the OM formalism(sec.2), which is the mathematical framework for our physical outlook embodied in the idea that the geometry and fields, with the internal symmetries and all interactions, as well the four major principles of relativity (special and general), quantum, gauge and colour confinement, are derivative. They come into being simultaneously in the stable system of the underlying “primordial structures” involved in the “linkage” establishing processes. The OM formalism is the generalization of secondary quantization of the field theory with appropriate expansion over the geometric objects leading to the quantization of geometry different from all existing schemes.

- We have chosen a simple setting and considered the primordial structures, which are designed to possess certain physical properties satisfying the general rules stated briefly in subsec.2.3, and have involved in the linkage establishing processes. The processes of their creation and annihilation in the lowest state (the regular structures) just are described by the OM formalism. In all the higher states the primordial structures are distorted ones, namely they have undergone the distortion transformations (subsec.2.4). These transformations yield the “quark” and “antiquark” fields defined on the simplified geometry (one  $u$ -channel) given in the subsec.2.4, and skeletonized for illustrative purposes. Due to geometry realization conditions held in the stable systems of primordial structures they emerge in confined phase. This scheme still should be considered as the preliminary one, which is further elaborated in the subsec.3.2 to get the physically more realistic picture.

- The distortion transformation functions are the operators acting in the space of the internal degrees of freedom (colours) and imply the incompatibility relations eq.(2.5.2), which hold for both the local and the global distortion rotations. They underly the most important symmetries such as the internal symmetries  $U(1)$ ,  $SU(2)$ ,  $SU(3)$ , the  $SU(2) \otimes U(1)$  symmetry of electroweak interactions, etc. (see sec.12, 17). We generalize the OM formalism via the concept of the OMM yielding the MW geometry involving the spacetime continuum and the internal worlds of the given number. In an enlarged framework of the OMM we define and clarify the conceptual basis of subquarks and their characteristics stemming from the various symmetries of the internal worlds. They imply subcolour confinement and gauge principle (subsec.3.2). By this we have arrived at an entirely satisfactory answer to the question of the physical origin of the geometry and fields, the internal symmetries and interactions, as well the principles of relativity, quantum, gauge and subcolour confinement. The value of the present version of hypothesis of existence of the MW-structures defined on the MW-geometry resides in solving of some key problems

of the SM, wherein we attempt to suggest a microscopic approach to the properties of particles and interactions.

- We derive the MW-SUSY (sec.8,9), which has an algebraic origin in the sense that it has arisen from the subquark algebra defined on the internal worlds, while the nilpotent supercharge operators are derived (sec.4). Therefore, the MW-SUSY realized only on the internal worlds but not on the spacetime continuum. Thus, it cannot be an exact symmetry of nature and has to be realized in its broken phase (sec.9). Our purpose above is much easier to handle, by restoring in the first the “exact” MW-SUSY. It can be achieved by lifting up each sparticle to corresponding particle state (sec.9). This enables the sparticle to be included in the same supermultiplet with corresponding particle. Due to different features of particles and sparticles when passing back to physically realistic limit eq.(9.11) one must have always to distinguish them by introducing an additional discrete internal symmetry, i.e., the multiplicative  $Z_2$   $R$ -parity (sec.9).

- We write then the most generic renormalizable MW-SUSY action eq.(11.17) involving gauge and supersymmetric matter frame fields, and, thus, the corresponding generating functional. Therein, we are led to the principal point of drastic change of the standard SUSY scheme to specialize the superpotential to be in such a form eq.(11.28)-eq.(11.34), which enables the microscopic approach to the key problems of particle phenomenology (sec.12).

- Within this approach, due to the symmetry of Q-world of electric charge the condition of realization of the MW connections has arisen embodied in the Gell-Mann-Nishijima relation (subsec.12.2). For the MW-structures the symmetries of corresponding internal worlds are unified into higher symmetry including also the operators of isospin and hypercharge. We conclude that the possible three lepton generations consist of six lepton fields with integer electric and leptonic charges being free of confinement condition (subsec.12.4). As well, the three quark generations exist composed of six possible quark fields (subsec.12.5), which carry fractional electric and baryonic charges realized in the confined phase. The global group unifying all global symmetries of the internal worlds of quarks is the flavour group  $SU_f(6)$  (subsec.12.3). The whole complexity of leptons, quarks and other composite particles, and their interactions arises from the MW-frame field, which has nontrivial MW internal structure and involves nonlinear fermion self-interaction of the components. This Lagrangian contains only two major free parameters, which are the coupling constants of nonlinear fermion and gauge interactions (subsec 12.6). To realize the MW-connections of the weak interacting fermions the P-violation compulsory occurred in W-world (subsec.12.8) incorporated with the gauge symmetry reduction. It has characterized by the Weinberg mixing angle with the fixed value at  $30^\circ$ . This gives rise to the local symmetry  $SU(2) \otimes U(1)$ , under which the left-handed fermions transformed as six independent doublets, while the right-handed fermions transformed as twelve independent singlets.

- Due to vacuum rearrangement in Q-world the Yukawa couplings arise between the fermion fields and corresponding isospinor-scalar  $H$ -mesons in conventional form. In the framework of suggested specific mechanism providing the effective attraction between the relativistic fermions caused by the exchange of the mediating induced gauge quanta in the W-world, the two complex self-interacting isospinor-scalar Higgs doublets ( $H_u, H_d$ ) as well as their spin- $\frac{1}{2}$  SUSY partners ( $\widetilde{H}_u, \widetilde{H}_d$ ) Higgsinos arise as the Bose-condensate. Taking into account this slight difference from the MSM arisen in the field content of MSMSM



in the Higgs sector the supersymmetric Lagrangian eq.(11.35) now also contains these fields coupled to the gauge fields in a gauge invariant way, when the symmetry-breaking Higgs bosons are counted off from the gap symmetry-restoring value as the point of origin (subsec.12.13).

- The Higgs mechanism does work in the following way: Before the symmetry was broken in the  $W$ -world, the 2 complex  $SU(2)_L$  Higgs doublets had 8 degrees of freedom. Three of them were the would-be Nambu-Goldstone bosons  $G^0$ ,  $G^\pm$ , which were absorbed to give rise the longitudinal modes of the massive  $W$ -components of the  $Z^0$  and  $W^\pm$  vector bosons, which simultaneously give rise the corresponding  $x$ - components too, leaving 5 physical degrees of freedom. The latter consists of a charged Higgs boson pairs  $H^\pm$ , a CP-odd neutral Higgs boson  $A^0$ , and CP-even neutral Higgs bosons  $h^0$  and  $H^0$ . The mass eigenstates and would-be Nambu-Goldstone bosons are made of the original gauge-eigenstate fields, where the physical pseudoscalar Higgs boson  $A^0$  is made of from the imaginary parts of  $h_u^0$  and  $h_d^0$ , and is orthogonal to  $G^0$ ; while the neutral scalar Higgs bosons are mixtures of the real parts of  $h_u^0$  and  $h_d^0$ . The mass of any physical Higgs boson that is SM-like is strictly limited, as are the radiative corrections to the quartic potential terms. We calculated the tree-level masses for these Higgs states (sec.16) and shown that the  $h^0$  Higgs boson arisen in the internal  $W$ -world is much heavier of that  $Z^0$  boson.

- In contrast to the SM, the suggested microscopic approach predicts the electroweak symmetry breakdown in the  $W$ -world by the VEV of spin zero Higgs bosons and the transmission of electroweak symmetry breaking from the  $W$ -world to the  $M_4$  spacetime continuum (subsec.12.14). The resulting Lagrangian of unified electroweak interactions of leptons and quarks ensues, which in lowest order approximation leads to the Lagrangian of phenomenological SM. In general, the self-energy operator underlies the Yukawa coupling constant, which takes into account a mass-spectrum of all expected collective excitations of bound quasi-particle pairs. If the MSM proves viable it becomes an crucial issue to hold in experiments the two testable predictions given in subsec.12.14.

- The realistic generating functional should be derived by passing back to the physical limit eq.(9.11). Such a breaking of the MW-SUSY can be implemented by subtracting back all the explicit soft mass terms formerly introduced for the sparticles eq.(15.2). These terms do not reintroduced the quadratic diagrams which motivated the introduction of SUSY framework. Therewith, the boson-fermion cancellation in the above-mentioned problems can be regarded as a consequence of a constraint stemming from holomorphy of the observables, therefore it will be held at the limit eq.(9.11) too. Thus, we extract the pertinent piece containing only the  $\eta$ -field components and then in afterwards pass to  $M_4$  (subsec.2.1) to get the final VMSM yielding the realistic particle spectrum.

- Thus, if the VMSM proves viable it becomes an crucial issue to hold in the experiments at LEP2 and at the Tevatron three testable solid implications given in sec.17, which are drastically different from those of conventional MSSM models.

- The implication of quarks into the VMSM is carried out in the same way of leptons except that of appearance of quark mixing with Cabibbo angle (subsec.17.1) and the existence of CP-violating complex phase in unitary matrix of quark mixing (subsec.17.2). The  $Q$ -components of the quarks contain at least one identical subquark, due to which the partial formfactors gain nonzero values. This underlies the quark mixing with Cabibbo angles. In lepton's case these formfactors are vanished and lepton mixing is absent. The CP-violation stems from the spanning eq.(17.2.2). Adopting a simple viewpoint on Higgs sector the masses of leptons and quarks are given in subsec.17.3.

We hope that the outlined VMSM, if it proves viable in the experiments at LEP2 and at the Tevatron, will be an attractive basis for the future theories. As yet no direct signal has been found in them, the absent of which has been cleared up the lower limits on Higgs bosons and sparticles masses.

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## Appendix A

### 1 The field equations

The state vectors are in the form

$$\begin{aligned}\chi^0(\nu_1, \nu_2, \nu_3, \nu_4) &= |1, 1 >^{\nu_1} \cdot |1, 2 >^{\nu_2} \cdot |2, 1 >^{\nu_3} \cdot |2, 2 >^{\nu_4}, \\ \nu_i &= \begin{cases} 1 & \text{if } \nu = \nu_i \text{ for some } i, \\ 0 & \text{otherwise,} \end{cases} \\ |\chi_-(1) > &= \chi^0(1, 0, 0, 0), \quad |\chi_+(1) > = \chi^0(0, 0, 0, 1), \quad < \chi_{\pm}(\lambda) | \chi_{\pm}(\mu) > = \delta_{\lambda\mu}, \\ |\chi_-(2) > &= \chi^0(0, 0, 1, 0), \quad |\chi_+(2) > = \chi^0(0, 1, 0, 0), \quad < \chi_{\pm}(\lambda) | \chi_{\mp}(\mu) > = 0, \end{aligned}$$

provided  $< \chi_{\pm} | A | \chi_{\pm} > \equiv \sum_{\lambda} < \chi_{\pm}(\lambda) | A | \chi_{\pm}(\lambda) > .$  The free field defined on the multimanifold  $G_N = G_{\eta} \oplus G_{u_1} \oplus \dots \oplus G_{u_N}$  is written

$$\Psi = \psi_{\eta}(\eta) \psi_u(u), \quad \psi_u(u) = \psi_{u_1}(u_1) \dots \psi_{u_N}(u_N),$$

where  $\psi_{u_i}$  is the bispinor defined on the internal manifold  $G_{u_i}$ . A Lagrangian of free field reads

$$\tilde{L}_0(D) = \frac{i}{2} \{ \bar{\Psi}_e(\zeta) {}^i \gamma^{(\lambda, \mu, \alpha)} \partial_i (\lambda, \mu, \alpha) \Psi_e(\zeta) - \partial_i (\lambda, \mu, \alpha) \bar{\Psi}_e(\zeta) {}^i \gamma^{(\lambda, \mu, \alpha)} \Psi_e(\zeta) \}. \quad (\text{A.1.1})$$

We adopt the following conventions:

$$\begin{aligned}
\Psi_e(\zeta) &= e \otimes \Psi(\zeta) = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \otimes \Psi(\zeta), \quad \bar{\Psi}_e(\zeta) = e \otimes \bar{\Psi}(\zeta), \quad \bar{\Psi}(\zeta) = \Psi^+(\zeta)\gamma^0, \\
{}^i\gamma^{(\lambda,\mu,\alpha)} &= {}^i\tilde{O}^{\lambda,\mu} \otimes \tilde{\sigma}^\alpha, \quad {}^i\tilde{O}^{\lambda,\mu} = \frac{1}{\sqrt{2}} \left( \nu_i \xi_0 \otimes \tilde{O}^\mu + \varepsilon_\lambda \xi \otimes {}^i\tilde{O}^\mu \right), \\
\varepsilon_\lambda &= \begin{cases} 1 & \lambda = 1 \\ -1 & \lambda = 2 \end{cases}, \quad < \nu_i, \nu_j > = \delta_{ij}, \quad \{ {}^i\tilde{O}^\lambda, {}^j\tilde{O}^\mu \} = \delta_{ij} {}^*\delta^{\lambda\mu}, \\
\tilde{O}^\mu &= \frac{1}{\sqrt{2}} (\xi_0 + \varepsilon_\mu \xi), \quad \tilde{O}^\lambda = {}^*\delta^{\lambda\mu} \tilde{O}_\mu = (\tilde{O}_\lambda)^+, \quad {}^i\tilde{O}^\mu = \frac{1}{\sqrt{2}} (\xi_{0i} + \varepsilon_\mu \xi_i), \\
\partial_{(\lambda,\mu,\alpha)} &= \partial / \partial {}^i\zeta^{(\lambda,\mu,\alpha)}, \quad \xi_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \xi = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \\
\xi_0^2 &= -\xi^2 = -\xi_{0i}^2 = \xi_i^2 = 1, \quad \{ \xi_0, \xi \} = \{ \xi_0, \xi_{0i} \} = \{ \xi_0, \xi_i \} = \\
&= \{ \xi, \xi_{0i} \} = \{ \xi, \xi_i \} = \{ \xi_{0i}, \xi_j \}_{i \neq j} = \{ \xi_{0i}, \xi_{0j} \}_{i \neq j} = \{ \xi_i, \xi_j \}_{i \neq j} = 0.
\end{aligned} \tag{A.1.2}$$

Field equations are written

$$\begin{aligned}
(\hat{p} - m) \frac{\psi(\eta)}{\eta} &= 0, \quad \bar{\psi}(\eta) \frac{(\hat{p} - m)}{\eta} = 0, \\
(\hat{p} - m) \frac{\psi(u)}{u} &= 0, \quad \bar{\psi}(u) \frac{(\hat{p} - m)}{u} = 0,
\end{aligned} \tag{A.1.3}$$

where

$$\begin{aligned}
\hat{p}_\eta &= i \frac{\partial}{\partial \eta}, \quad \hat{p}_u = i \frac{\partial}{\partial u}, \quad \hat{u} = {}^i\gamma^{(\lambda\alpha)} \frac{\partial}{\partial u_i} \partial_{(\lambda\alpha)}, \quad \frac{\partial}{\partial (\lambda\alpha)} = \partial / \partial \eta^{(\lambda\alpha)}, \quad \frac{\partial}{\partial u_i} \partial_{(\lambda\alpha)} = \partial / \partial u_i^{(\lambda\alpha)}, \\
{}^i\gamma^{(\lambda\alpha)}_\eta &= {}^i\tilde{O}^\lambda_\eta \otimes \tilde{\sigma}^\alpha = \nu_i \xi_0 \otimes \gamma^{(\lambda\alpha)} = \nu_i \xi_0 \otimes \tilde{O}^\lambda \otimes \tilde{\sigma}^\alpha, \\
{}^i\gamma^{(\lambda\alpha)}_u &= {}^i\tilde{O}^\lambda_u \otimes \tilde{\sigma}^\alpha = \xi \otimes {}^i\gamma^{(\lambda\alpha)} = \xi \otimes {}^i\tilde{O}^\lambda \otimes \tilde{\sigma}^\alpha, \\
(\gamma^{(\lambda\alpha)})^+ &= {}^*\delta^{\lambda\tau} \delta^{\alpha\beta} \gamma^{(\tau\beta)} = \gamma_{(\lambda\alpha)}, \quad \left( {}^i\gamma^{(\lambda\alpha)}_u \right)^+ = -{}^i\gamma_{(\lambda\alpha)}_u.
\end{aligned} \tag{A.1.4}$$

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